Earth-Centered Earth-Fixed
Scalable Visualization without Distortion

Noel Zinn
Hydrometronomics LLC
SWIGGIS at PBX Systems
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www.hydrometronomics.com
Hydrometronics LLC

Hydrometronics provides consultancy and technical software development for seismic navigation, ocean-bottom positioning, subsea survey, geodesy, cartography, **3D visualization (ECEF)** and wellbore-trajectory computation.

The scope of Hydrometronics' offerings is due to the long career of its principal (click on the 'about us' link above). Hydrometronics is primarily a Matlab® shop, providing compiled, user-friendly, GUI-driven applications or .NET DLLs that solve client problems in all the fields cited above, which are not exclusively nautical. Click on the 'contact us' link above to initiate a conversation about your needs.

In addition to software development, Hydrometronics consults in all these fields, bringing a career of expertise to bear on your needs.

For a sampler of the disciplines addressed by Hydrometronics, visit the 'downloads' link above for papers presented over the years or visit the four heritage links below for extractions from those papers (preserved in place for the web bots).

**EGM08 in ECEF (above)**

Earth Gravitational Model 2008 (EGM08) is the best, world-wide, freely-available model of the geoid. Earth-Centered Earth-Fixed (ECEF) is the 3D, orthogonal, geocentric, Cartesian, coordinate system used by GPS, which empowers distortion-free visualization. Geoidal undulations are exaggerated 10,000 times here for visual effect.
EGM2008·10,000 in ECEF
Overview

• Cartography (2D) is distorted.
• Geodesy (3D) is not, but …
• … 3D visualization environment (VE) required,
• … and geoid also required.
• Coordinate Reference System (CRS) primer
• Earth-Centered Earth-Fixed (ECEF)
• Topocentric coordinates (a “flavor” of ECEF)
• Orthographic projection (topocentric in 2D)
• This presentation => www.hydrometronics.com
Cartography is distorted …

Globular projection

Orthographic projection

Stereographic projection

Mercator projection
... but geodesy is not distorted

ECEF in a VE
ESRI ArcGlobe
Issues

• Plate-to-pore scalability is desired in earth science software
• That software has heretofore used 2D projected coordinates in the horizontal and 1D depth/time in the vertical
• Projections have distortions of linear scale, area and azimuth that increase with project size
• These distortions can be quantified and managed on an appropriate map projection
Issues

• Earth science software is evolving toward visualization environments (VEs) that:
  – Operate in a 3D “cubical” CRS
  – Excel at graphical manipulation
  – Are geodetically unaware

• A pure 3D approach will:
  – Exploit the native power of VEs
  – Avoid the distortions (3D=>2D) of map projections
  – Achieve plate-to-pore scalability
  – Provide a new perspective on the data
What are some VEs?

- Gocad (Paradigm, proprietary)
- Petrel, HueSpace (Schlumberger, proprietary)
- Matlab (The Mathworks, proprietary)
- ArcScene (ESRI, proprietary)

- VTK (Visualization Toolkit, open source)
- Mayavi (Python GUI front end to VTK, open source)

- iPod/Phone/Pad? Android? (some day, if not already!)
Heritage earth-science applications with internal geodesy support any projected coordinate system (2D horizontal + 1D vertical), but with the usual, well-known mapping distortions.
Current Path to VE via Middleware

VEs have no internal geodesy. Coordinates are projected “outside the box” (in middleware). Only one coordinate system is allowed inside the box at a time.
If ECEF coordinates are chosen in middleware, the VE “sees” the world in 3D without any mapping distortions. If ECEF coordinates in WGS84 are chosen, then projects throughout the world will fit together seamlessly.
EPSG Coordinate System Primer

1. **Geographical 2D** (lat/lon) and **Geographical 3D** (lat/lon/height with respect to the ellipsoid)
2. **Vertical** (elevation or depth w.r.t. the geoid)
3. **Projected** (mapping of an ellipsoid onto a plane)
4. **Engineering** (local “flat earth”)
5. **Geocentric Cartesian** (Earth-Centered Earth-Fixed)
6. **Compound** (combinations of the above)
Geographical CS: lat/lon/(height)

A graticule of curved parallels and curved meridians (latitudes and longitudes) intersect orthogonally on the ellipsoid. Height is measured along the normal, the straight line perpendicular to the ellipsoid surface.
Vertical CS: elevation

Elevation is measured along the (slightly curved) vertical, which is perpendicular to the irregularly layered geopotential surfaces of the earth. The geopotential surface at mean sea level is called the geoid. (Graphic from Hoar, 1982.)
Projected CS: Northing/Easting

• Map projections of an ellipsoid onto a plane preserve some properties and distort others
  – **Angle** - and local shapes are shown correctly on conformal projections
  – **Area** - correct earth-surface area (e.g., Albers)
  – **Azimuth** - can be shown correctly (e.g., azimuthal)
  – **Scale** - can be preserved along particular lines
  – **Great Circles** - can be straight lines (Gnomonic)
  – **Rhumb Lines** - can be straight lines (Mercator)

• **Rule of thumb**: map distortion $\propto$ distance$^2$
Projected CS Distorts

Globular projection

Orthographic projection

Stereographic projection

Mercator projection

Rule of thumb: map distortion \( \propto \text{distance}^2 \)
Engineering CRS (“Flat-Earth”)

Our project extracted from an ellipsoidal earth

Our project extracted from a cubical, flat earth
The Z-axis extends from the geocenter north along the spin axis to the North Pole. The X-axis extends from the geocenter to the intersection of the Equator and the Greenwich Meridian. The Y-axis extends from the geocenter to the intersection of the Equator and the 90E meridian.
Coordinate Conversion

• The mathematics of map projections (3D=>2D) are complicated (especially TM) and generally valid over limited extents.

• The mathematics of converting Geographical CS coordinates to ECEF Geocentric CS (3D=>3D) are simple and valid the world over.
So, Why ECEF?

- ECEF is the geodetic CS native to a 3D VE
- ECEF in a 3D VE is a globe in your hands
- Given the proper perspective (turning the globe), ECEF coordinates have no distortion
- ECEF is scalable from plates to pores
- No geodetic “smarts” are required in the VE
North America in VTK
U.S.G.S. Coastline Culture
Excerpts in Geographical and ECEF

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Translation & Rotation: ECEF to Topocentric

• A journey back to Projected CS because …
• … some users may prefer their data referenced to their local area of interest
• ECEF can easily be translated and rotated to a topocentric reference frame
• This conversion is conformal, it preserves the distortion-free curvature of the earth, and the computational burden is small
• VEs already do something similar to change the viewing perspective
EPSG Graphic of Topocentric

Greenwich meridian
\( \lambda = 0 \)

Geocentric CRS origin
\( \varphi = 0, \lambda = 0, h = a \)
\( X = Y = Z = 0 \)

Topocentric CS origin
\( \varphi = \varphi_0, \lambda = \lambda_0, h = h_0 \)
\( X = X_0, Y = Y_0, Z = Z_0 \)
\( U = V = W = 0 \)
GOM in Topocentric Coordinates
Topocentric to Orthographic

- Continuing the journey
- The orthographic projection is the view from space, e.g. our view of the moon
- Topocentric without the W vertical coordinate (3D=>2D) is the Orthographic projection
- The ellipsoidal Orthographic projection is a bona fide map projection with quantifiable distortions intermediate between our usual 2D+1D paradigm and a new topocentric / ECEF 3D paradigm
Orthographic Projection of the Moon
Orthographic Projection of GOM

Graticule on Orthographic

Easting

Northing

Hydrometronics LLC
Oblique Ellipsoidal Orthographic Minimum Scale

This is scale in the radial direction. Scale in the circular direction is 1.0000
Our Journey Schematically

1 / Proj

All the undistorted curvature of the Earth in a 3D VE
Our Journey Schematically

Proj 2D+1D
Northing
Easting
Elevation

Geog 2D+1D
Latitude
Longitude
Elevation

EGM
Geog 3D
Latitude
Longitude
Height

Orthographic
Northing
Easting
(Vertical)

U=E, V=N

Topocen 3D
U
V
W

Δ + Θ

Geocen 3D
X
Y
Z

All the undistorted curvature of the Earth in a 3D VE
Summarizing

- Cartography (2D) is distorted; geodesy (3D) is not
- Not all 3D presentations are ECEF (geodesy)
- Geodetically “unaware” visualization environments (VE) present an opportunity
- Coordinate Reference System (CRS) primer
  - Earth-Centered Earth-Fixed (ECEF)
  - Topocentric coordinates (a “flavor” of ECEF)
  - Orthographic coordinates (2D topocentric)
Conclusion

• The real world is 3D
• New visualization environments are 3D
• Why incur the distortions of a 2D map projection entering real-world data into a VE?
• ECEF, topocentric and orthographic coordinates are a paradigm shift in the way we view our data, a perspective that may extract new information
• It’s time for ECEF!
More Information

- This presentation can be downloaded at www.hydrometronics.com
- There is a ECEF Group on LinkedIn
- Guidance Note 7-2 at www.epsg.org
- Wikipedia (search ECEF)
- World coastlines are available at www.ngdc.noaa.gov/mgg/shorelines/shorelines.html
Extra Slides
Mini Bio of Noel Zinn

• Noel Zinn began Hydrometronics LLC in 2010 as a technical software consultancy
• Geodesist for ExxonMobil in the Naughties
• Navigation Scientist for Western Geophysical in the Nineties
• Surveyor for NCS International in the Eighties
• Navigator for Delta Exploration (Singapore) in the Seventies
• Peace Corps Volunteer in India in the Sixties
• Studied at the University of California (Berkeley) and the University of Houston
Geographical to ECEF Coordinates

Given the ellipsoid semi-major axis \((a)\) and flattening \((f)\), and latitude \((\phi)\), longitude \((\lambda)\), and height \((h)\)

\[
b = a - a \cdot f \quad e^2 = \frac{(a^2 - b^2)}{a^2} \quad \nu = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}
\]

\[
X = (\nu + h) \cos \phi \cos \lambda \\
Y = (\nu + h) \cos \phi \sin \lambda \\
Z = (\nu(1 - e^2) + h) \sin \phi
\]
ECEF to Geographical Coordinates

Given ellipsoid $a$ and $f$, and $X$, $Y$ and $Z$ Cartesians

\[
\begin{align*}
    b &= a - a \cdot f \\
    e^2 &= (a^2 - b^2)/a^2 \\
    e'^2 &= (a^2 - b^2)/b^2 \\
    \nu &= \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}} \\
    p &= (X^2 + Y^2)^{1/2} \\
    \theta &= \tan^{-1} \left( \frac{Z \cdot a}{p \cdot b} \right) \\
    \phi &= \tan^{-1} \left( \frac{Z + e'^2 b \sin^3 \theta}{p - e^2 a \cos^3 \theta} \right) \\
    \lambda &= \tan^{-1} \left( \frac{Y}{X} \right) \\
    h &= \left( \frac{p}{\cos \phi} \right) - \nu
\end{align*}
\]
# U.S.G.S. Coastline Culture Excerpts in ECEF and Topocentric

## Geocentric CRS (ECEF)

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### U.S.G.S. Coastline Culture Excerpts in Topocentric and Orthographic

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