Ocean-Bottom Cable Detector Positioning: Acoustics versus First Breaks

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Abstract
Due to various geophysical advantages mentioned in this paper, ocean bottom cable (OBC) seismic surveys are gaining popularity in water depths down to 150 metres and perhaps much deeper in the future. Today geodesists and geophysicists are debating the issue of OBC detector positioning. This paper compares and contrasts the advantages and disadvantages of two common methods: independent acoustic surveys and the use of first breaks (energy onset) collected during the seismic survey itself. It explains that sub-meter results are possible with both systems, but that first-break positioning can be more reliable than acoustics. Two appendices offer (A) a brief description of the SDCÔORD algorithm for positioning detectors using first breaks and (B) a glossary that defines important geodetic terms.

1 Introduction

Fig. 1: Layout of an OBC survey showing two lines of cables with detectors on the bottom, the recording vessel in the centre and the source vessel shooting orthogonally to the cables.

Ocean-bottom cable seismic surveying is well described in the popular literature (Barr and Scott, 1997, and Rayson, 1997). Figure 1 shows the important elements of an OBC survey. There are cables with dual seismic sensors (hydrophones and geophones) connected to a recording vessel in the middle of the graphic. The shooting vessel with one or two air gun arrays that produce the seismic energy is shown sailing a regular pattern orthogonal to the cables. Orthogonal shooting has geophysical and positioning advantages, but in-line shooting is also possible. The collected midpoints of all possible combinations of sources and detectors comprise a swath of coverage. Figure 2 shows the back deck of a cable laying vessel with automated cable-handling gear. Some of the advantages of OBC over conventional streamer surveys are the flexibility of acquisition geometry, greater surface consistency (i.e., more combinations of source and detector at different azimuths for a given midpoint, useful for resolving static delays and for amplitude compensation), more flexibility in working around obstructed zones, the use of dual sensors to remove ghosts and layer reverberations (Barr and Sanders, 1989), reduced noise due to the elimination of cable vibration and strumming due to towing and surface weather conditions, and better coverage due to the elimination of cable feather caused by currents.

Source positioning in OBC is similar in technique and quality to source positioning in deep-water streamer surveys. It basically consists of Global Positioning System (GPS) receivers on the source array. On the other hand, detector positioning techniques are less-widely standardized in OBC than in land or deep-water streamer surveys. Three techniques are common in the industry: (1) recording and using the drop coordinates of the detectors, (2) deploying high-frequency acoustic sensors attached to the detectors and positioned by a pinging survey usually independent of the seismic survey and (3) using multiple occasions of the onset of seismic energy (first breaks) as surveying observations in a positioning algorithm. A combination of acoustics and first breaks is also possible.

2 Overview of Positioning Methods

Since drop positions must be recorded anyway to assure that the actual detector location bears some resemblance to the planned location, this technique is the cheapest and easiest to implement. In shallow water the detector drop position can be close to the resting position. But, in deeper water, this is not likely due to currents and drop trajectories. Consequently, drop coordinates are not analysed further in this paper, although they have a role in very shallow water.

High-frequency acoustic systems are provided by several vendors for OBC (First Break, October 1997). This equipment is technologically similar to that used in streamer surveys for years. Acoustics provide a precise observable, essentially a time break computed in hardware. Consequently, acoustic surveys can be quite accurate within the limitations of systematic errors that are detailed below. Unfortunately, acoustic positioning is expensive (extra equipment) and it may be operationally time consuming. Limiting the expense (e.g., interpolating detector coordinates between fewer acoustic sensors) or the time (e.g., pinging less) has a profound impact on the precision and reliability of detector coordinates when using acoustics.
Seismic first-breaks can be picked by any number of automated methods that choose a significant change in the amplitude or inflection of the arriving seismic energy. The trace can be preconditioned by band-pass filtering or by deconvolution to improve signal-to-noise ratio. Automated picking can be enhanced with neural networks, or especially troublesome picks can be made by hand (facilitated by computer displays). Figure 3 shows a raw seismic trace with the first break noted. (Compression waves drive the seismic sensor negatively.)

The time of a first-break pick can be related to distance. Distances can be processed in a positioning algorithm. First-break positioning potentially combines the cost advantages of drop positions with the accuracy of acoustics. In a seismic survey, the marginal cost of picking and processing first breaks is low since the personnel, software and seismic data are already on the job. Although each first break is a crude observable by navigation standards, we enjoy an abundance of observations, especially when refracted energy is processed. (In fact, not using refracted energy sometimes wastes useful information. This loss may degrade quality control representatives must be aware of them all.

4.1 Random Error
Due to their high frequency (over 30 kHz), acoustic positioning systems have small random error, equivalent to one to three decimetres at one standard deviation. First-break random pick quality varies from prospect to prospect as a function of geology and sampling interval in the amount of 2 to 6 milliseconds. Depending upon the velocity of propagation (water or refraactor), this converts to 3 to 10 metres expressed as a standard deviation. Since lower-frequency seismic energy (under 250 Hz) is not so severely attenuated by refraction through the earth as high-frequency acoustics, we have a long offset range of useful first-break picks. Many picks reduce the effect of pick random error on detector coordinates. For example, given typical methods of execution (described in Section 4.17), you may have 40 successful pings into an acoustic sensor, but 1,000 first breaks into a seismic detector. According to Gauss's law of random error propagation this generates a 5 to 1 advantage for first breaks (the square root of the ratio 1000/40), presuming that the azimuthal distributions are equally balanced.

4.2 Source Location
Acoustic pingers are often co-located on vertical arms with a GPS antenna. First-break energy originates from the seismic energy source itself. A seismic company's entire navigation infrastructure is brought to bear to confirm valid source positions. Both acoustic and first-break source positions are subject to similar, occasional deterioration due to poor GPS DOP or loss of differential corrections. These tend to affect acoustic sensor positions more than first-break detector positions simply because fewer source positions are involved in the former than the latter. Although acoustic and first-break source positions are fixed, source positioning is actually an option with first breaks under circumstances of adequate source DOP and otherwise good detector positions. This feature can actually enable the seismic crew to avoid reshoots.

4.3 Seismic Array Size
For direct arrivals through the water, the gun responsible for the onset of energy will be the gun closest to the detector. Given the dimensions and orientation of the source array perimeter, the coordinates for the source centre and the azimuth of the energy path from source to detector, all known factors, approximate gun...
coordinates (the intersection of pick azimuth and the array perimeter) can be computed. If this computation is not made, and geometry is balanced, this systematic error will average out and propagate into the predicted random error of the detectors.

In the case of refracted energy, this computation is less relevant. Refracted seismic energy will group around the source centre due to the coalescence of the air bubble and the critical angle of refraction determined by Snell's Law. In many prospects, most picks will be refracted.

4.4 Instrumental Delay

Any time delay in associating GPS coordinates with the time of an acoustic ping can introduce a bias that must be correctly compensated in software. These delays may be well-quantified between the seismic energy and primary GPS receiver, but often a different receiver is used in the acoustic system. Some receivers offer more than one data stream: one raw, one filtered. Which one is being used? All these issues must be known and quantified in the acoustic software to assure competent acoustic results.

With first breaks, instrument delay between the recorded start of cycle and the actual firing of the source can be a systematic error. Presuming that this delay is common to the entire prospect, it is removed in the global polynomial regression stage of the SDCOORD algorithm (Appendix A.1). Similar bias modelling is not currently available in acoustic processing software.

4.5 Definition of the onset of energy

Different first-break pickers, or different picking techniques within the same picker, will have different mathematical definitions of the onset of seismic energy. Like instrumental delay, this systematic error common to all picks is removed in the global polynomial regression.

4.6 Detector depth

Depth corrections are required to reduce slant ranges to their horizontal component in acoustic and first-break systems alike. Depending on water depth and horizontal offset, a depth error of 1 metre can easily cause a horizontal error of 1 metre or more for a given range. A depth error of 5 metres can lead to big trouble. Some acoustic software does solve for depth. But like height in GPS, depth in an acoustic survey is a poorly-resolved parameter due to vertical dilution of precision (VDOP, mentioned in the Glossary). Due to an incorrect calibration of the fathometer, an incorrect knowledge of the velocity of acoustic propagation in water or to silt on the ocean floor, depth readings may be globally biased. In the case of SDCOORD-processed first breaks, such a bias is mitigated by the global polynomial regression. In general, detector-depth errors are mitigated by a good distribution of azimuths and by a preponderance of far offsets. Both conditions are more likely with first breaks than with acoustics.

4.7 Velocity of propagation in water

Acoustic systems measure time that must be converted to a distance by multiplication by the acoustic velocity of propagation in water, a quantity usually measured near the surface. But velocity usually varies with depth. Even if velocity-profile probes are deployed and their results used in the acoustic software, velocity may also vary laterally over the prospect. Also, thermal layers may bend signals. Sometimes thermal layers may be impenetrable by high-frequency acoustics. An error in the velocity of propagation will act like a scale factor bias on the acoustic range, lengthening or shortening it. Guessing incorrectly at the velocity of propagation can be disastrous. With first breaks this issue is dealt with by modelling the vertical velocity gradient, discussed next.

4.8 Vertical Velocity Gradient

A vertical velocity gradient is a variation in velocity as an increasing function of offset. This occurs because the longer-offset picks are increasingly likely to have arrived via deeper, faster refractors. Deeper layers are not always faster than shallower layers, but if they are not faster they will not carry the first arrival. Figure 4 illustrates a vertical gradient. It shows a single source event and the many paths the seismic energy may take to arrive first at each detector. Detectors near the source will see the energy first directly through the water, which may vary in velocity as a function of depth. Detectors farther from the source will see the energy first through faster refractive layers. We can use all this information in an appropriately-modelled positioning algorithm. The method in SDCOORD is the global polynomial regression (discussed in Appendix A.1).

4.9 Lateral Velocity Gradient

A lateral (or horizontal) velocity gradient is a variation in velocity as a function of position in a geological field. A lateral velocity gradient behaves like scale factor in what cartographers refer to as a conformal map projection. It may be caused, for example, by a greater compaction of sedimentation as one moves farther offshore. Since the refracted energy used in OBC first-break positioning primarily travels through the recent sedimentary layers, a lateral velocity gradient may sometimes be a factor in positioning results. Lateral velocity gradients (if they exist) are modelled in SDCOORD by source-specific and detector-specific velocity trends (Appendix A.2).

4.10 Inadequate geometry or number of pings

Some systematic errors like instrumental delay and poorly-known depth are mitigated by observations at opposing azimuths and offsets, and they should be! Systematic errors will have their maximum effect when the geometry or the number of observations is inadequate. Acoustic pinging is time consuming. Operational considerations may mandate an expedited acoustic survey (e.g., rapidly sailing by the detectors) or surveying only one side of the line. Rapid sailing, one-sided pinging and acoustic sensors masked by the cable or other obstructions cause inadequate geometry and too few pings. On the other hand, operational considerations always assure the best possible seismic data from which first breaks are derived.
4.11 Multi-path (surface ghosts)
Dual-sensor technology (hydrophones and geophones together) decrease the effects of surface ghosts in OBC seismic data. Similar technology is not yet implemented for acoustic sensors. Consequently, multipath can create blunders in acoustic positioning.

4.12 Vessel Noise
Engine and propeller noise from the pinging vessel affect high-frequency acoustic systems much more than lower-frequency seismic energy. Blunders are created.

4.13 Muddy bottoms
Some prospects may have muddy silt into which the acoustic sensor may sink. Masked acoustic signals may result. By contrast, muddy bottoms enhance dual-sensor coupling for first-break seismic energy.

4.14 Moving Detectors
Detectors dropped on the sea floor may sometimes move due to currents or vessels, fortunately not often. Detectors positioned by acoustic pinging before the seismic survey may become mispositioned detectors during the seismic survey. Since first-break positioning uses the seismic data itself, the window for misadventure is smaller.

4.15 Administration
Associating acoustic sensors with the seismic detectors with which they are coupled can be an administrative nightmare. With current technology this is often done by hand-recorded notes. Some acoustic software relates positioned sensors with the pre-plotted detector position. This is successful in shallow water, but is often incorrect in deep water. Technology is rapidly improving in this area with the use of radio frequency identification (RFID) tags and acoustic sensors embedded in the seismic cable, but it remains a source of potential mistakes. On the other hand, associating seismic energy with the right source and detector is a routine part of seismic processing.

4.16 Complex near-surface geology
Complex near-surface geology is a catch-all phrase for geological conditions not modelled in the first-break positioning algorithm, such as gas bubbles caused by decaying alluvial sediments, which may affect refracted energy. The first-break strategy for dealing with a complex near-surface geology is to process the widest-possible offset range. When we are confident that under normal conditions (without a complex near-surface geology) near and far offsets produce statistically-equivalent results, and we shoot over, through, under and around near-surface geological anomalies (if they exist) with a wide offset range, we mitigate their potential effect on our final coordinates.

4.17 Susceptibility to Blunder
Figure 5 shows a typical in-line acoustic pinging geometry with a 4 kilometre cable and a break in one of the pinging lines, which are offset 100 metres on either side of the receiver line. Acoustic ranges are limited to 500 metres (an extreme range for such systems) in the preanalyses (see Glossary) that follow. All possible pings within this offset are used, from a low of 10 per detector on the east end, to 40 per detector through most of the cable, and down to 20 on the west end since the pinging lines do not overlap the receiver line. Actual acoustic yield will often be less than this for several of the reasons cited above.

Figure 6 plots the unitless DOP and MEEM values for the 80 detectors. The DOP value of 0.4 for most of the line means that if the acoustic random error is, in fact, a decimetre, then the expected precision will be 0.04 metres DRMS. This is an exhilarating number and the basis for much of the confidence in acoustics. Of course, it presumes that any systematic errors are perfectly modelled and solved in the positioning software. The MEEM value, also about 0.4 for most of the line, means that the positional shift due to the single worst undetected, unrejected blunder will also be 0.04 metres. Again, for reasons discussed above, there may be more than one such blunder. We shouldn’t be comfortable when the MEEM is nearly equal or greater than the DOP. It means that our reported precision is not reliable (in the technical surveying sense explained in the Glossary), i.e., not accurate within the reported precision. Notice that MEEM is significantly larger than DOP in the far east of this line. Poor geometry and too few pings have a profound effect on the reliability of acoustic positions.

Figure 7 shows a typical orthogonal seismic shooting geometry with 16 detectors (every sixth in this swath subset) and more than 2,500 shots. This survey was shot around obstructions, an excellent use of OBC. For the preanalyses that follow, first breaks are limited to a source-detector offset of 1200 metres, typical for this technique using refracted energy, although longer offsets are possible. All possible first breaks within this offset are used,
ranging from 916 to 1070 per detector. First-break yield is often this high.

![Fig. 7: Plan view of an orthogonal OBC survey showing 16 selected detectors and 2500 shots. Units of easting and northing are meters.](image)

Figure 8 plots the unitless DOP and MEEM values for the 16 detectors. The DOP value of 0.06 for most of the line means that if first-break random error is, in fact, 3 metres then the expected precision will be 0.18 metres DRMS. Not too shabby! Of course, it also presumes that any systematic errors are perfectly modelled and solved in the positioning software. The MEEM value of 0.01 means that the positional shift due to the single worst undetected, unrejected blunder will be 0.03 metres. Again, there may be more than one such blunder. But in the case of first breaks, we can be confident when the MEEM is significantly less than DOP. It means that our reported precision is reliable and accurate. The phenomenal redundancy available with first breaks not only reduces random error, but increases resistance to blunder and, by extension, to bias.

![Fig. 8: Plots of unitless DOP and MEEM for the 16 detectors in Figure 7.](image)

5 Conclusion

As ocean bottom cable surveys have gained popularity in the seismic industry, both geophysicists and geodesists debate the issue of detector position. Because acoustics have been standard equipment in deep-water seismic and hydrographic surveys for years, both are comfortable with them despite their expense. On the other hand, some geodesists are not yet comfortable with first-break picking techniques or the fact that this observable is propagated through a strange new medium, the earth itself. Most geophysicists know about first-break picking and the earth, but some may not appreciate that this observable and this medium can be modelled in a precise positioning algorithm. Numerous comparisons between these techniques in production demonstrate that this is the case.

This paper has described the advantages and disadvantages of both acoustic and first-break methods. It has shown that first-break positioning is a viable, reliable, cost-effective alternative to acoustics for ocean bottom cable surveys.

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Appendix A: SDCOORD

Source-Detector Coordinate Recomputation (SDCOORD) is a Western Geophysical, Omega™ Seismic Function Module that positions detectors with first-break picks. SDCOORD is a sequential least-squares algorithm based upon the measurement model of an extended Kalman filter. A global polynomial regression pre-processor relates pick time to distance in three dimensions and solves for several systematic errors. Blunder rejection is implemented in the global polynomial regression stage.
with a simple, but effective, difference tolerance. Computed source-specific and detector-specific velocity trends model a lateral velocity gradient if it exists. Figure 9 is a flow diagram of SDCOORD.

A.1 Global Polynomial Regression
In SDCOORD, our observations are our picks. A pick is associated with nominal coordinates and depths for the source and the detector. The distance (or offset) between these nominal coordinates can be determined using the Pythagorean Theorem in three dimensions. Using least squares, offsets are regressed against pick times to determine the coefficients of the best-fitting polynomial of a user-selected order. Using this polynomial, a “regressed” distance in metres or feet is determined for a specific pick in milliseconds by substitution into the polynomial. After some additional steps (see velocity trends below), these regressed distances are processed by the measurement model.

Figure 9 shows computed distances in metres plotted against pick times in milliseconds for a subset of a swath. The best-fitting, fifth-order polynomial is also plotted as seen at both ends of the data points, but is obscured elsewhere. This polynomial determines the relationship between pick times and regressed distances that are processed by the measurement model. Notice that the polynomial does not cross the origin of the plot. This static, y-axis intercept at zero pick time absorbs the systematic errors of instrumental delay and delay (or anticipation) in the mathematical definition of the onset of seismic energy in the first-break picker. The shape of the polynomial at near offsets will correct for a global bias in bathymetry by bending into an approximate hyperbola. The polynomial differentiated with respect to pick time (i.e., its slope) quantifies the vertical velocity profile over the refracted offset range. Outlying picks are shown on this plot. They are rejected with a difference tolerance with respect to the polynomial. This improves coordinate results and predicted coordinate error (DRMS).

A.2 Velocity Trends
After the global polynomial regression, one or two optional types of least-squares regressions can be performed, one type for every source gather and one type for every detector gather, as many regressions as there are total sources and detectors (i.e., tens of thousands for a typical swath). These regressions produce source-specific velocity trends and detector-specific velocity trends (scale factors near unity). When implemented, the geometric mean of the velocity trends for the source and detector associated with each pick is multiplied by the globally regressed distance before processing in the measurement model. By providing a varying field of scaling factors over all sources and detectors in the prospect, velocity trends model a lateral velocity gradient, if it exists.

A.3 Measurement Model
The equations of the measurement model of a sequential, extended Kalman filter are widely published in the literature (Gelb, 1974). SDCOORD rigorously adheres to this algorithm for the processing of regressed, velocity-trended distances from which modelled biases and blunders have been removed as described above.

A.4 Iteration Until Convergence
All stages of the SDCOORD algorithm are iterated in sequence until coordinate convergence to some user-defined tolerance is achieved.

A.5 Quality Control
SDCOORD provides a wealth of quality control statistics by which to evaluate results. These include the unscaled coordinate variances in the grid axes, the unit variance factors (UV, see Glossary) for all the detectors, the DRMS (radial error) values scaled by the UV, the number of rejected picks, the total number of used picks for each detector and the number in each quadrant, heuristic “cross-correlations” that reduce the octant distribution of geometry to single numbers for quick reference, the sum of all residuals in each quadrant, the coefficients of the best-fitting polynomial, all the velocity trends and the distance to the adjacent detector. Plots of residuals as a function of pick time can be produced. Additionally, the usual seismic quality control of linear moveout corrected first arrival displays can be produced.

Appendix B: Glossary of Selected Terms
Accuracy
Classically, accuracy is defined as conformance with a standard, the closeness of an estimated value to an accepted value of some quantity. In geodesy this standard is usually a well-defined reference system of coordinates. Unfortunately, this definition begs the question. If the “conformance” is known, accuracy is absolute. Precision is the degree of refinement of a measurement or, by extension, to the coordinates that are a least-squares function of those measurements. Of course, blunders and biases are assumed to have been removed. Since reliability quantifies the effects of blunder and, to some extent, bias, accuracy is today defined in terms of precision and reliability (i.e., DRMS and MEE). See separate entry for reliability.

Adjustment
Process of deriving corrections (residuals) to measured or computed quantities to compensate for random error. The least-
squares adjustment criterion stipulates that the sum of the squares of the residuals be a minimum.

Bias
Bias is also called systematic error. Biases are generally-consistent, often small, one-sided, systematic deviations from the truth that can, if known, be expressed mathematically and solved as part of the adjustment. Examples of bias are an instrumental delay, which can be subtracted from every observation, or a change in velocity of energy propagation, which can be multiplied by every observation. Systematic errors in observations will propagate into systematic errors in coordinates without being represented in the predicted coordinate uncertainty computed by random error propagation. This is undesirable. The best solution for systematic errors is to identify them and model them functionally, that is, to solve for them. Fortunately, we usually have lots of first-break redundancy in OBC to solve for biases, if only we are creative in our modelling techniques.

Blunder
Blunders are also called outliers or spikes or mistakes. Generally, they are large, occasional and unpredictable. In the days of hard-recorded, conventional surveying they may be caused by inadvertently transposing recorded numbers or by sighting the wrong target. In our electronic era they may be caused by multipath (acoustic reflections from the surface), dropped bits in data communication or unpredictable irregularities in the travel path (like gas pockets in decaying alluvial sedimentation). Blunders are handled by outlying-observation rejection schemes (called data snooping) that may be simple or sophisticated. The most mathematically-developed scheme is the Delft Method, which quantifies the resistance of a given survey to blunders in terms of the marginally detectable error (MDE) and the marginal error effect (MEE).

Delft Method (also known as the B Method)
Data snooping technique that uses the unscaled $w$-statistic to identify outliers in an adjustment. The worst outlier is eliminated, the adjustment recomputed and the residuals tested again. The B-Method is named for the Dutch geodesist at Delft Technological University, W. Baarda.

Dilution of Precision (DOP)
Dilution of precision is a unitless measure of how effectively the precision of a measurement propagates into the precision of computed coordinates. It is a measure of geometry and the number of observations. Technically speaking, it is the square root of the sum of selected elements on the diagonal of the inverse of the unweighted normal coefficient matrix. Components of DOP are GDOP (Geometric DOP) that measures X, Y, Z and time in GPS, PDOP (Position DOP) that measures X, Y and Z, HDOP (Horizontal DOP) that measures X and Y, in-line DOP and cross-line DOP, and VDOP (Vertical DOP) that measures only the vertical component. Because of four GPS dimensions (X, Y, Z and time), a limited number of satellites and unbalanced geometry (all satellites are above, none below), GPS DOPs are rarely less than 1. In horizontal systems (like OBC) with balanced, all-sided geometry, HDOP can be significantly less than 1. The DOPs reported in this paper are HDOPs. Multiply range random error by DOP to get DRMS.

DRMS
DRMS is distance root mean squared or radial error, the square root of the sum of the positional variances in any number of orthogonal axes, usually two. There is a 63% to 68% probability that the least-squares estimate of detector position lies within a circle of 1DRMS centered at its true position. The circle of radius 2DRMS represents 95% to 98% probability. The variation in probability has to do with the correlation between the uncertainties in the two orthogonal axes and their relative size.

Error ellipse
An error ellipse is an ellipse of positional uncertainty defined by two, uncorrelated, orthogonal axes, one maximal (the semi-major axis) and one minimal (the semi-minor axis), and their orientation. The parameters of the error ellipse are derived from the positional variances in X and Y and their covariance.

Extended
An extended Kalman filter is one whose non-linear measurement (or dynamic) model is linearised about the predicted state. A linearised Kalman filter is linearised about the previous state.

Iteration
Iteration is the successive repetition of a mathematical algorithm, using the result of one stage as the input for the next.

Kalman filter
The Kalman filter is an optimal estimation technique in space and time that is a recursive solution to the least-squares problem. Named after R. E. Kalman who published his work in 1960 and 1961.

Least Squares
Least-squares estimation is a method of solving over-determined systems of observation equations by imposing the constraint that the sum of the squares of the observational residuals must be a minimum. The method was independently invented by Gauss in 1795 and Legendre in 1806.

Marginally Detectable Error (MDE)
Also called internal reliability, the MDE is the smallest observational blunder (or bias) that can be detected with the $w$-statistic given a significance and power of the hypothesis test. For the preanalyses in this paper the power is 80% and the significance is 0.27%.

Marginal Error Effect (MEE)
Also called external reliability, the MEE is the hypothetical shift in position induced by the blunder that is marginally detectable and unrejected by the employed outlier rejection scheme (i.e., the MDE).

Marginal Error Effect Multiplier (MEEM)
Unitless MEEM is the normalized MEE, the MEE divided by the observational standard deviation. Multiply range random error by MEEM to get MEE.

Preanalysis
Given the number of observations and their geometry in a planned survey, certain quality measures can be computed even before the survey begins. When the observations are ranges only (acoustic or first-break, but not azimuths, hyperbolas, etc.), DOP and MEEM can be preanalysed.

Random Error
Unlike counting (which is discrete, i.e., based on integers), every observation (or measurement, which is continuous, i.e., based on real numbers) contains random error. Random error is the sum of all the small, unpredictable variations inherent in the
physical process of making an observation with the tools available. Random error is two-sided and tends to zero mean (i.e., to "average out") over a sequence of observations. Random error can be decreased as instrument technology (precision and resolution) improves and its effects can be minimized by a good geometrical distribution and increased number of observations, but random error cannot be eliminated. In a weighted least-squares adjustment the random error of the observations is propagated into the predicted random error of the coordinates by statistical laws developed by the mathematician C. F. Gauss. Observational random error is quantified in terms such as standard deviation. Coordinate random error is described by terms such as DRMS (distance root mean square, or "radial error") or by the semi-major and semi-minor axes and orientation of the error ellipse. The number of ranges and their geometrical content is quantified by Dilution of Precision (DOP). Range random error multiplied by DOP equals DRMS. Because the term error has the connotation of being bad, which should only apply to avoidable biases and blunders, we sometimes refer to unavoidable random error as uncertainty.

Reliability
Internal and external reliability (MDE and MEE) are measures of the resistance of a computed position to blunder and bias. Reliability is a statistical product of the Delft Method of data snooping.

Residual
A residual is the difference between an actual observation and its adjusted value.

Uncertainty
Uncertainty is a neutral expression of random error, usually the standard deviation.

Unit Variance Factor
The unit variance factor is a measure of the "fit" of an adjustment given observation uncertainty. In the case of uncorrelated observations, it is the sum of the squares of the normalized residuals (the residual divided by the observational standard deviation) divided by the degrees of freedom in the adjustment (the number of observations less the number of coordinates, i.e., two, X and Y).

w-Statistic
The w-statistic, simply stated, is the ratio of an observation's residual divided by the computed standard deviation of that residual. The w-statistic may be normally distributed or tau distributed depending upon the validity of our knowledge of observational uncertainty and whether or not we've scaled by the unit variance factor.