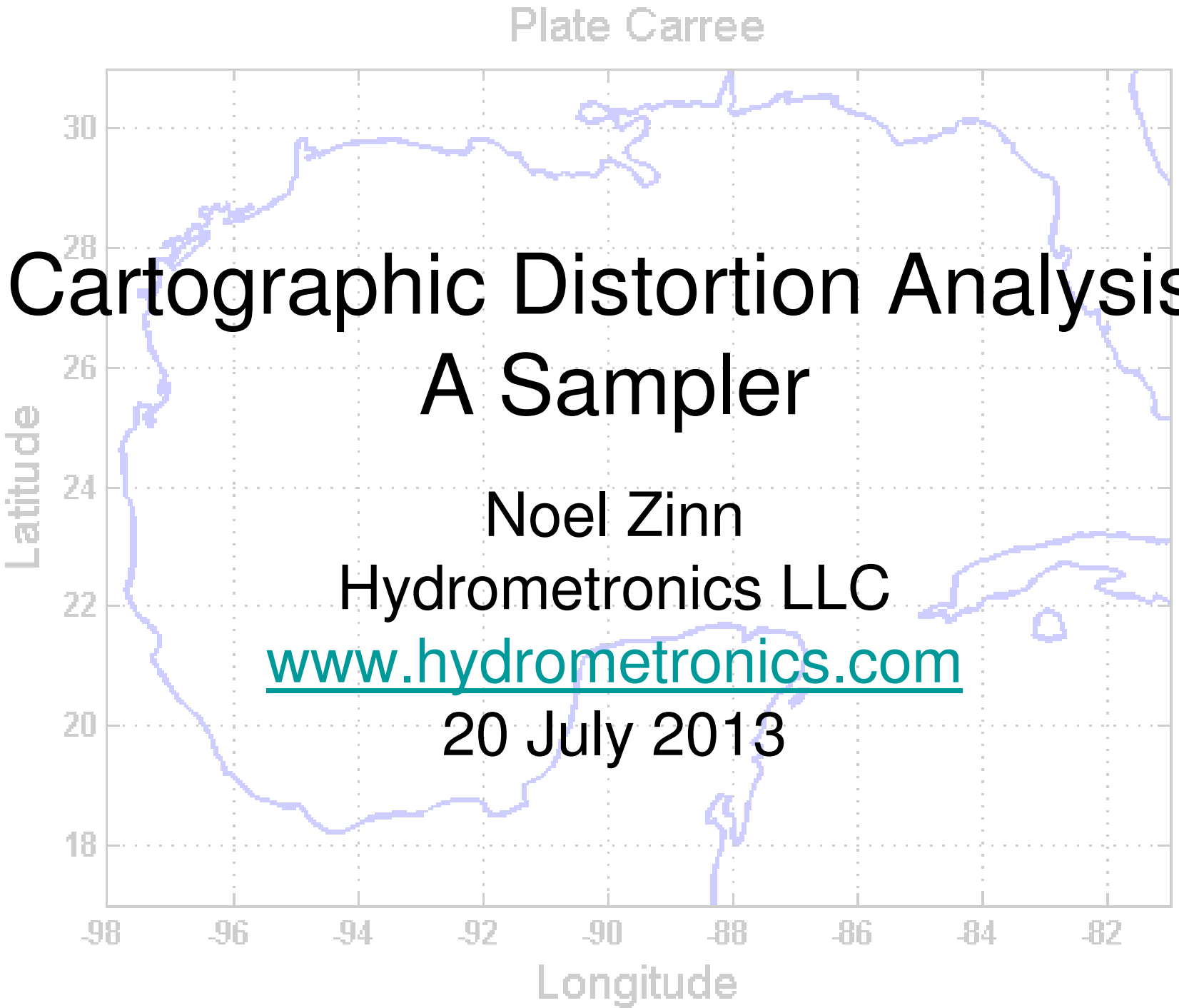



# Cartographic Distortion Analysis: A Sampler

Noel Zinn
Hydrometronics LLC
www.hydrometronics.com

20 July 2013

# Overview

- A seismic prospect may be surveyed in a pre-defined local projection.
- In the US that might be State Plane coordinates on land or UTM in US Survey Feet in the outer continental shelf of the Gulf of Mexico.
- Local prospects may be gathered regionally or a regional survey may be undertaken.
- In these cases new parameters or a different projection may be required to minimize cartographic distortion over the region.
- Cartographic distortion analysis is the process of computing and plotting distortion contours on a map containing the prospect.
- It is also the process of optimizing projection parameters to minimize distortion in the area of interest (AOI).
- Among conformal projections, e.g. Mercator, Transverse Mercator (TM), Oblique Mercator (OM), Lambert Conformal Conic (LCC) and Stereographic, the two main distortions are (1) point scale factor and (2) convergence of the meridians.
- Two non-conformal projections are treated in this report, the Albers Equal Area and the Ellipsoidal Orthographic (EO).

The defining characteristic of a conformal projection is that the scale factor at a point is uniform in all directions. It is for this reason that angles (not azimuths) in the real world are the same on the map. The point scale factor is the differential ratio of grid distance to geodetic distance.

Azimuths (or directions) in the real world (true azimuths) and azimuths on the map (grid azimuths) will differ in the amount of convergence of the meridians. At a point, convergence is the same in all directions on conformal projections.

On equal-area projections the point scale factor is not the same in all directions. In fact, the factors are reciprocal in orthogonal directions. The product of reciprocal scale factors is unity (1), thus the projection is equal area. Angular distortion is characteristic of an equal-area projection.

The ellipsoidal orthographic is neither conformal nor equal-area, but it has other advantages. It is derived from topographic (ENU) and Earth-Centered Earth-Fixed (ECEF) coordinates.

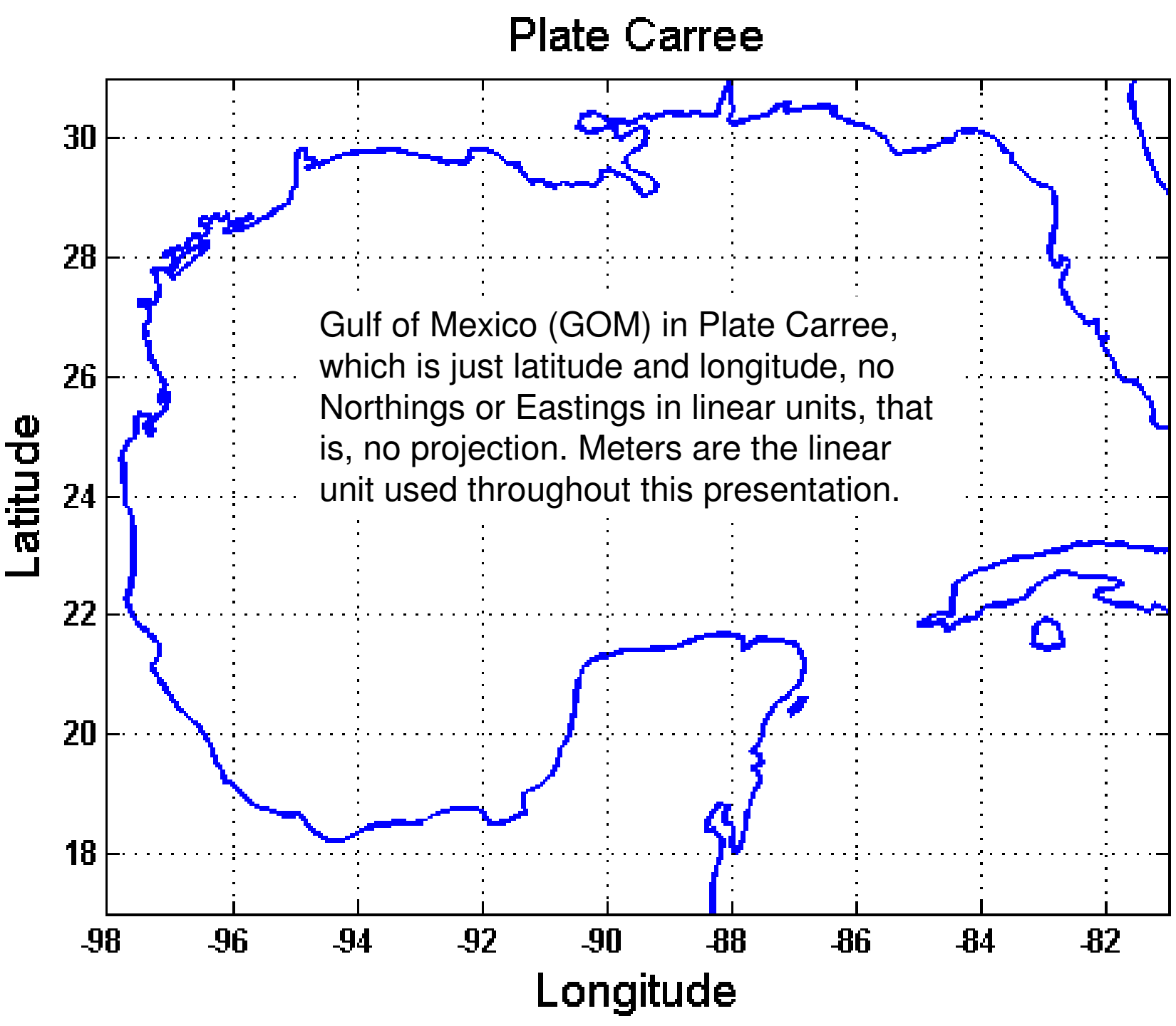
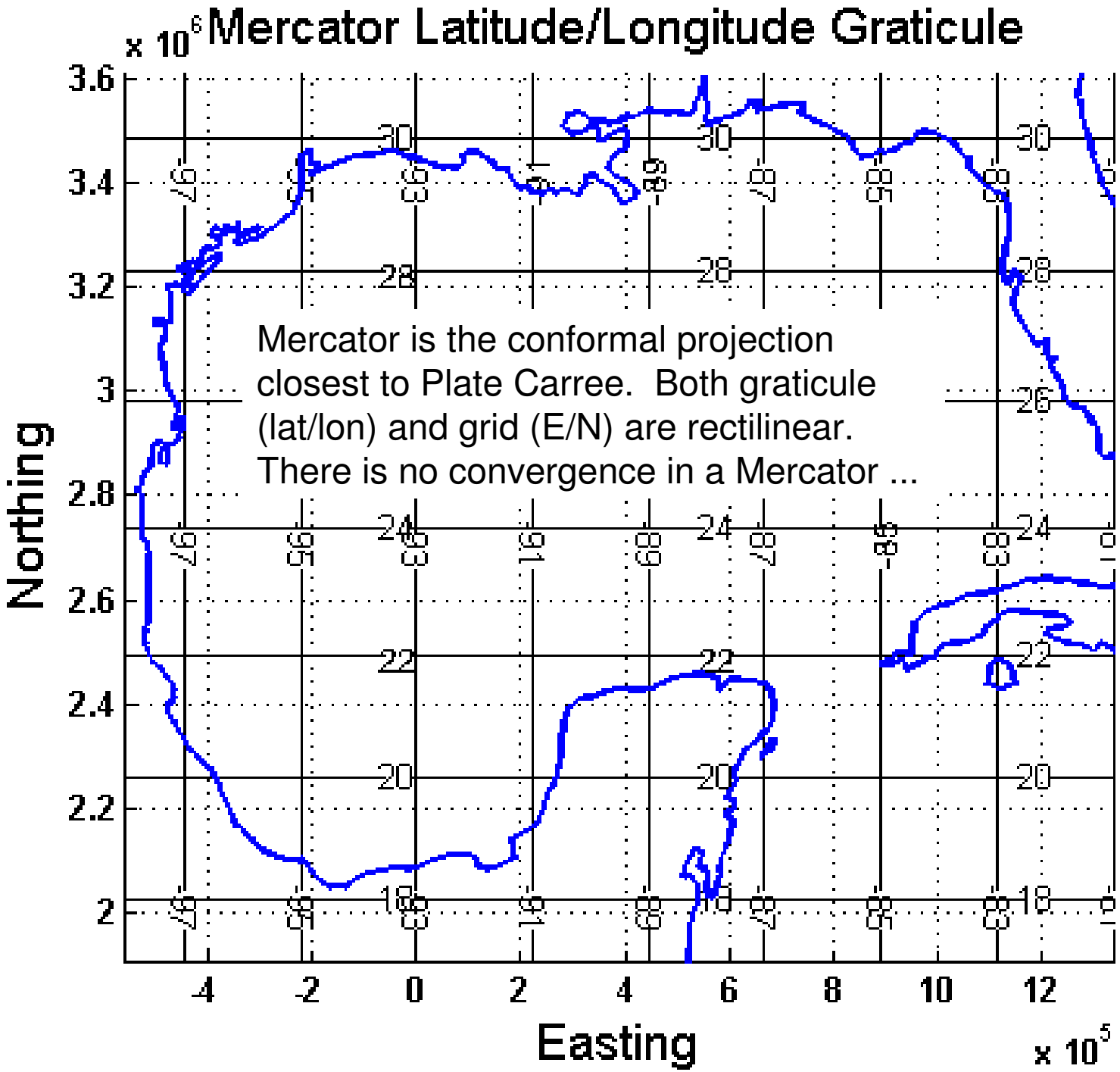
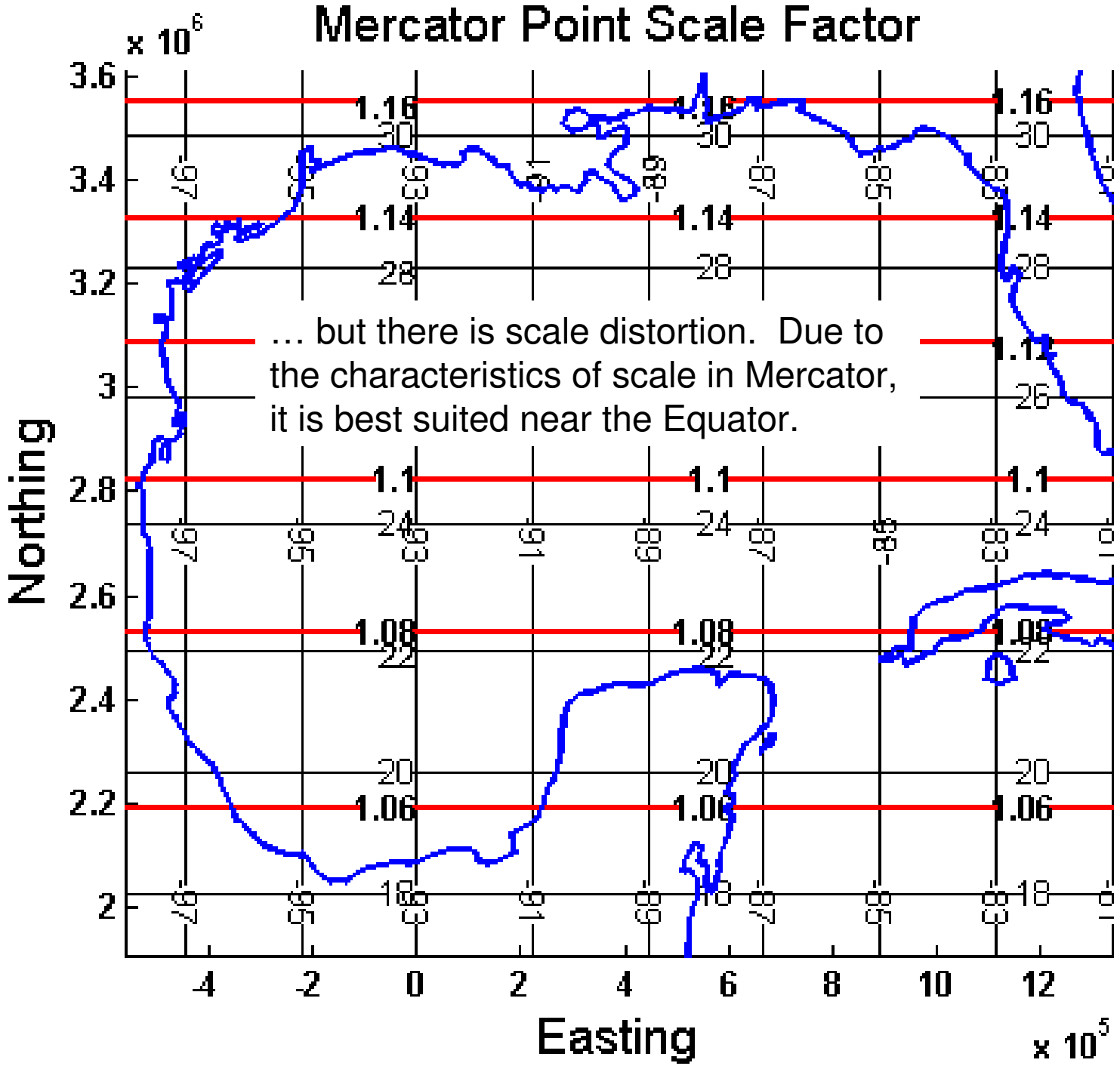
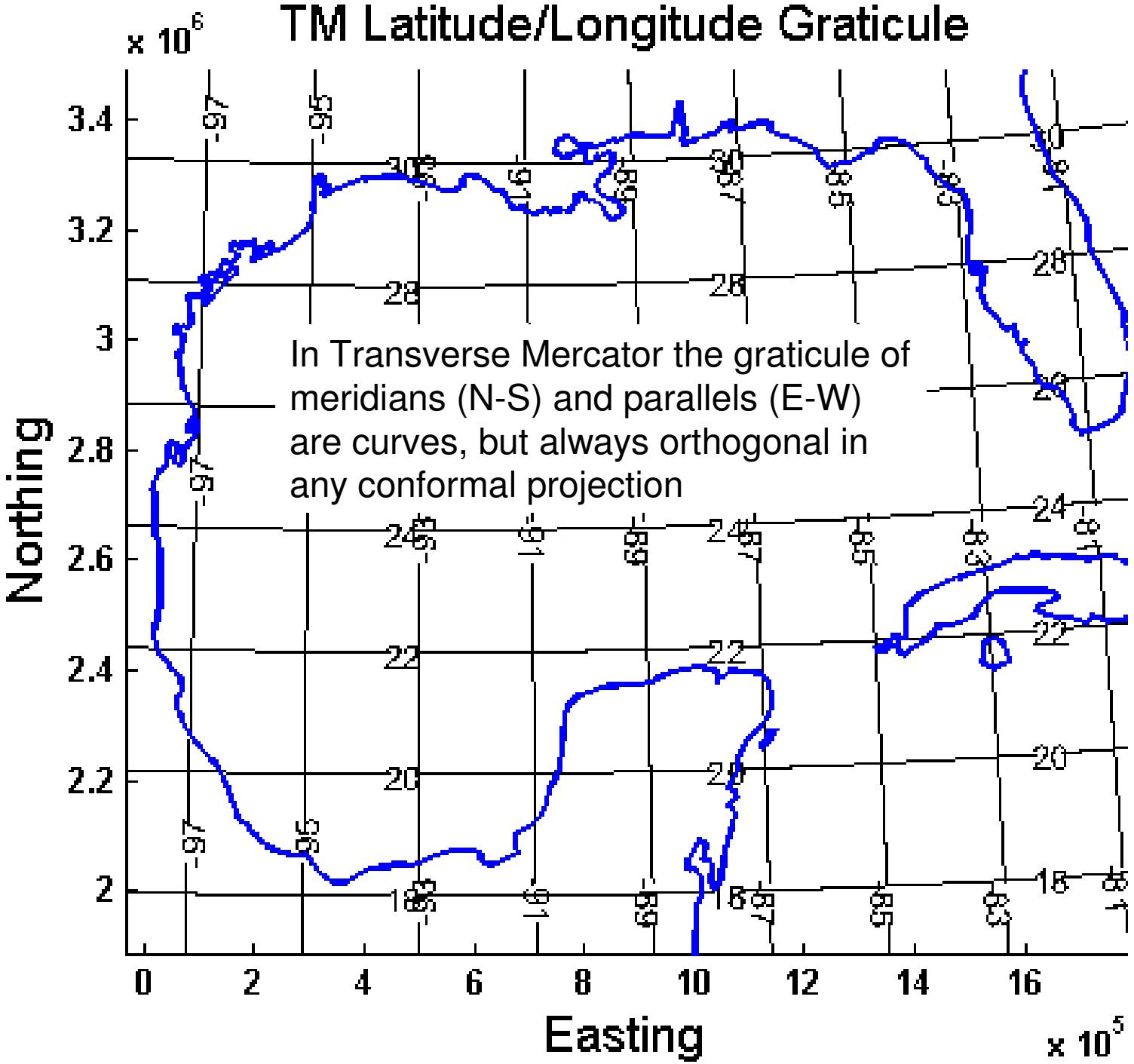
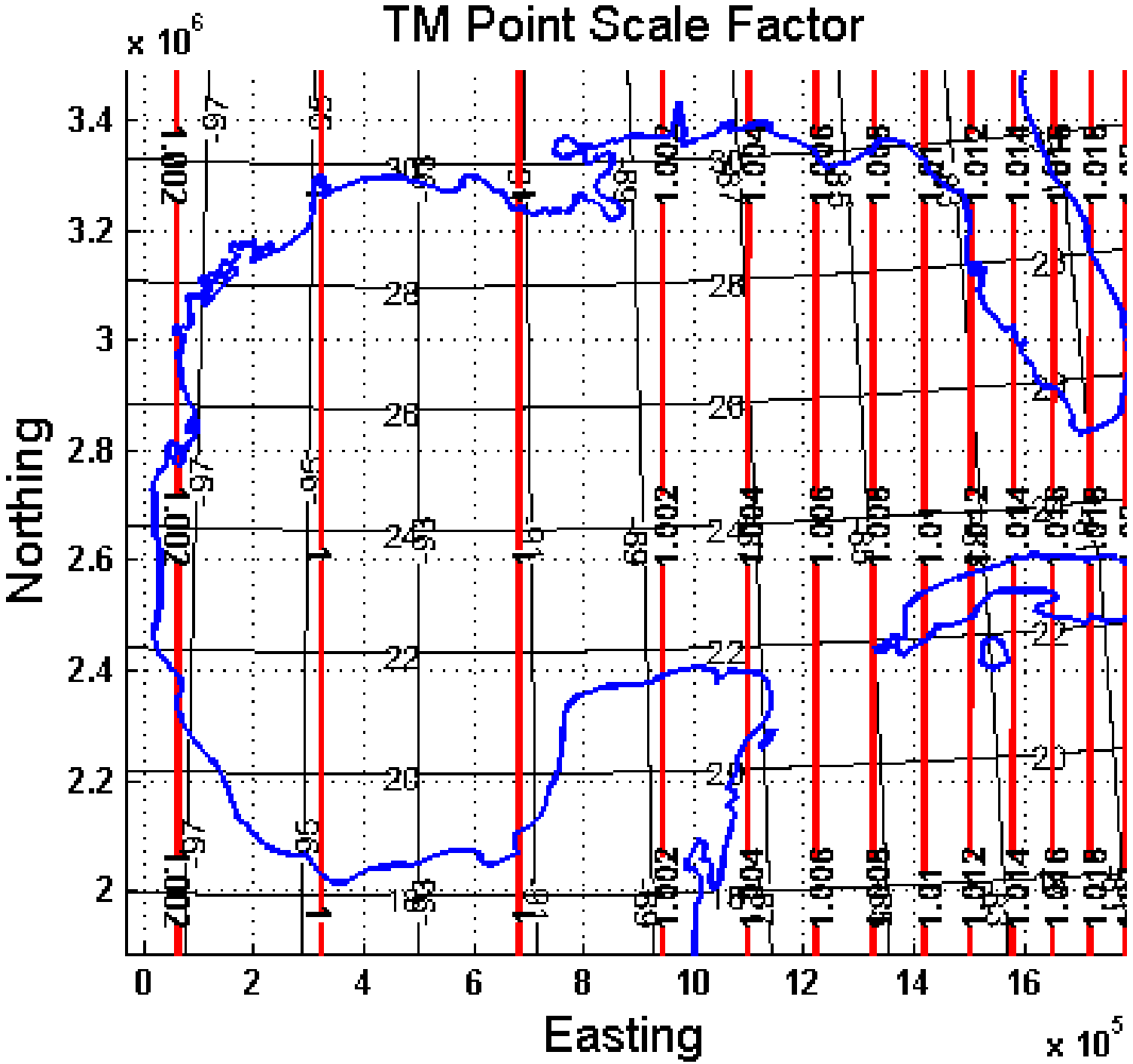


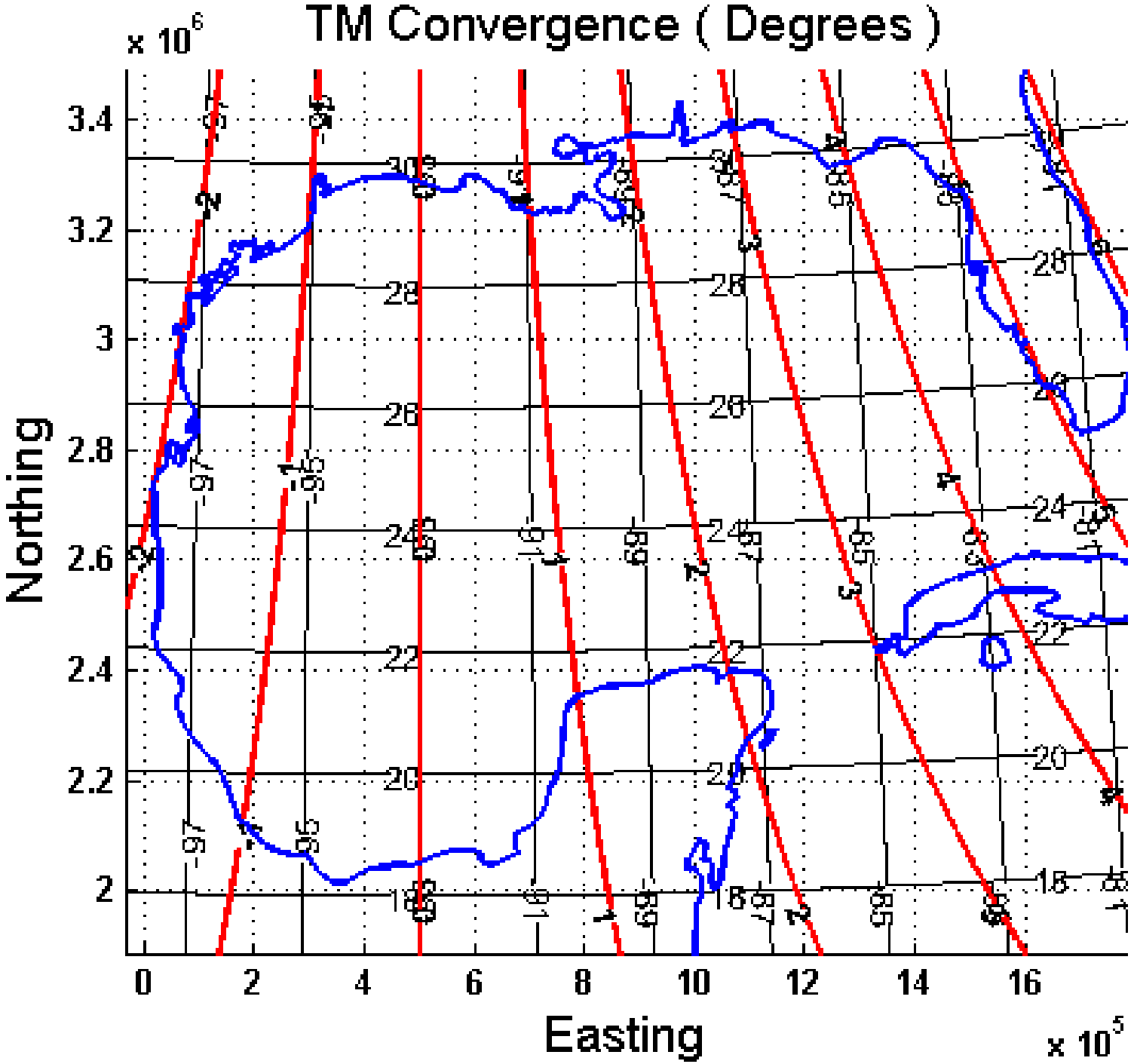
Plate Carree
Gulf of Mexico (GOM) in Plate Carree, which is just latitude and longitude, no Northings or Eastings in linear units, that is, no projection. Meters are the linear unit used throughout this presentation.
Latitude
30
28
26
24
22
20
18
-98
-96
-94
-92
-90
-88
-86
-84
-82
Longitude

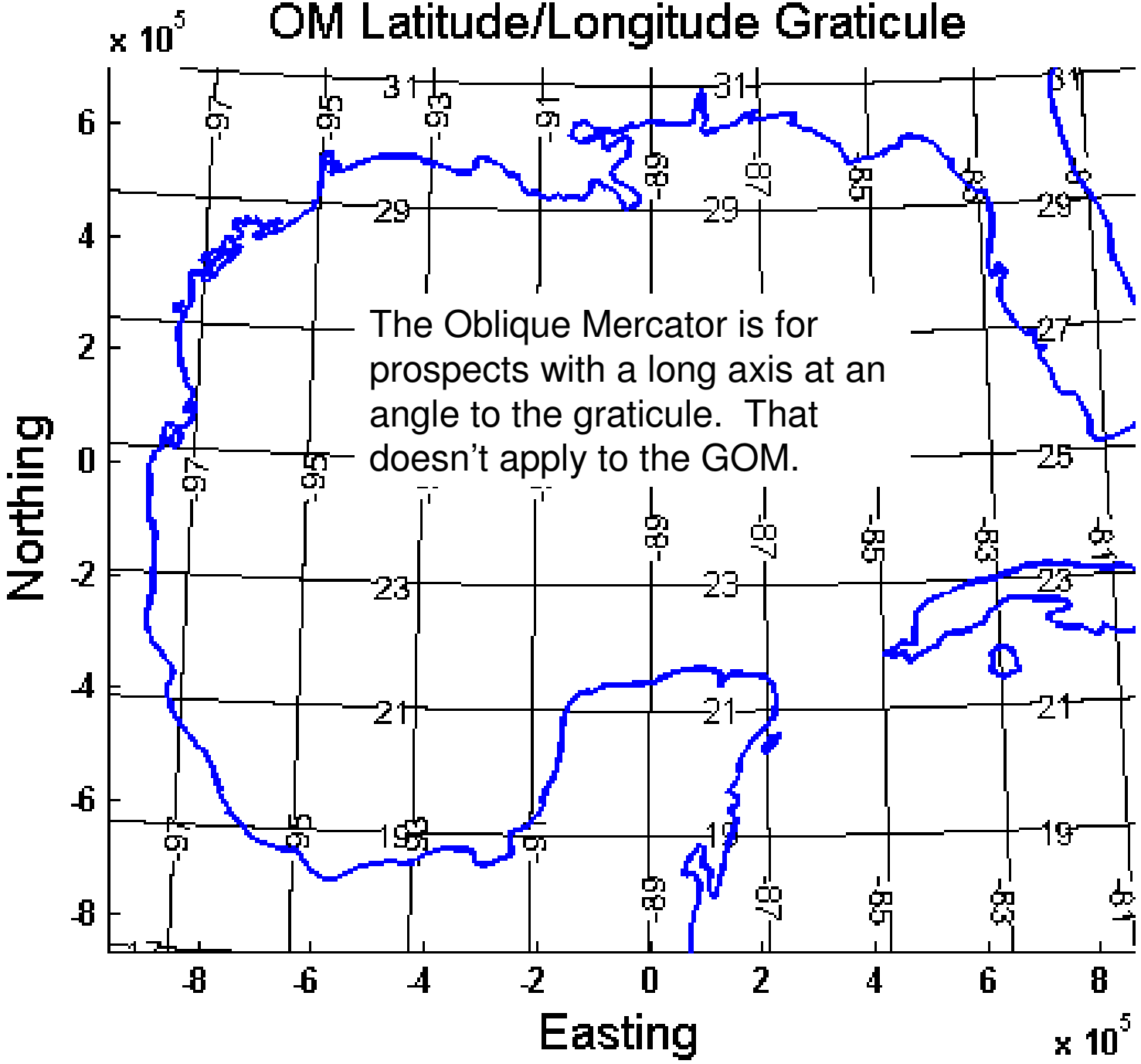

Mercator Latitude/Longitude Graticule
x 10^6
Northing
Easting
x 10^5
Mercator is the conformal projection closest to Plate Carree. Both graticule (lat/lon) and grid (E/N) are rectilinear. There is no convergence in a Mercator ...

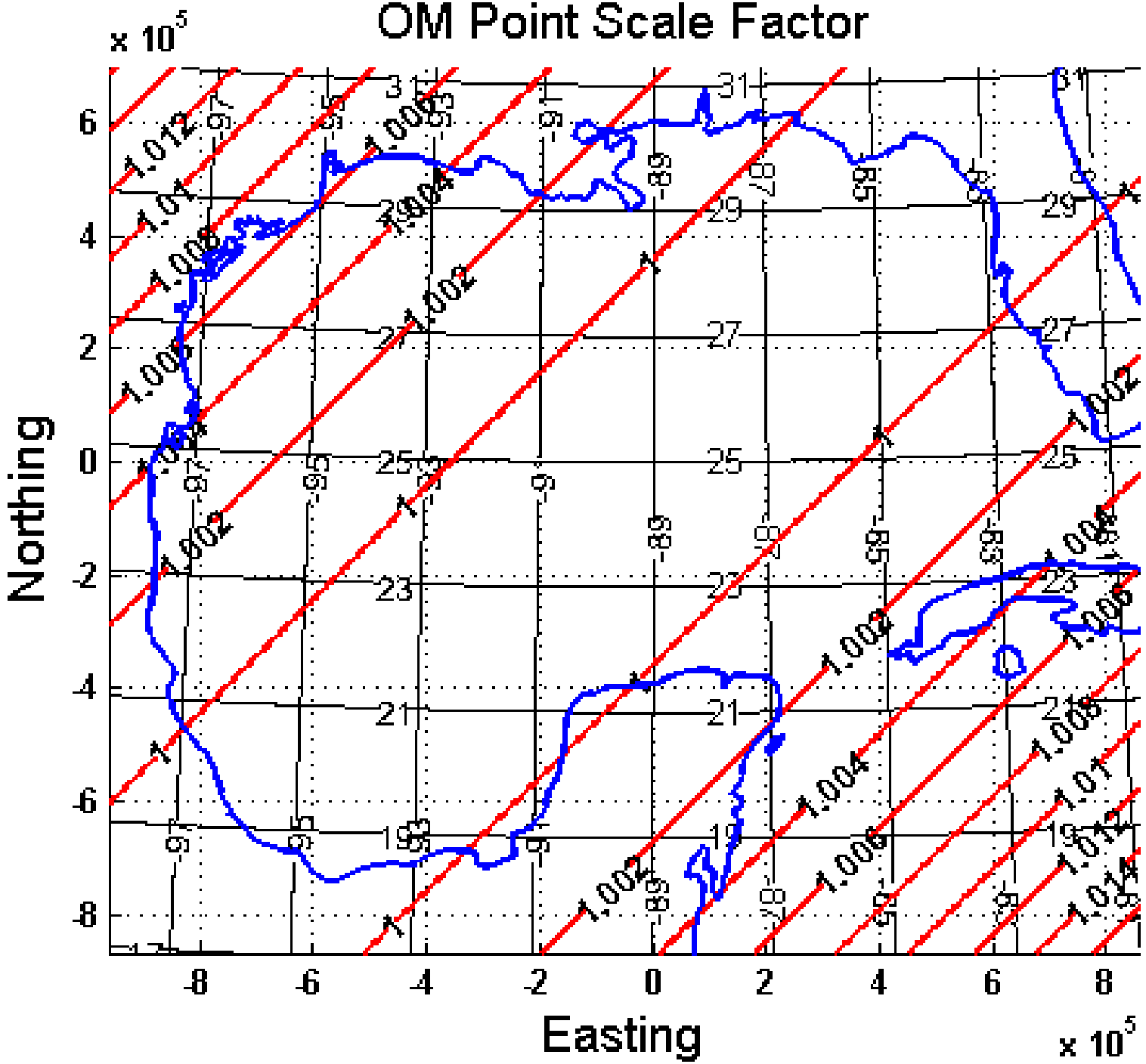

Mercator Point Scale Factor
x 10^6
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Northing
1.16
1.14
1.12
1.1
1.08
1.06
30
28
26
24
22
20
18
… but there is scale distortion. Due to the characteristics of scale in Mercator, it is best suited near the Equator.
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5

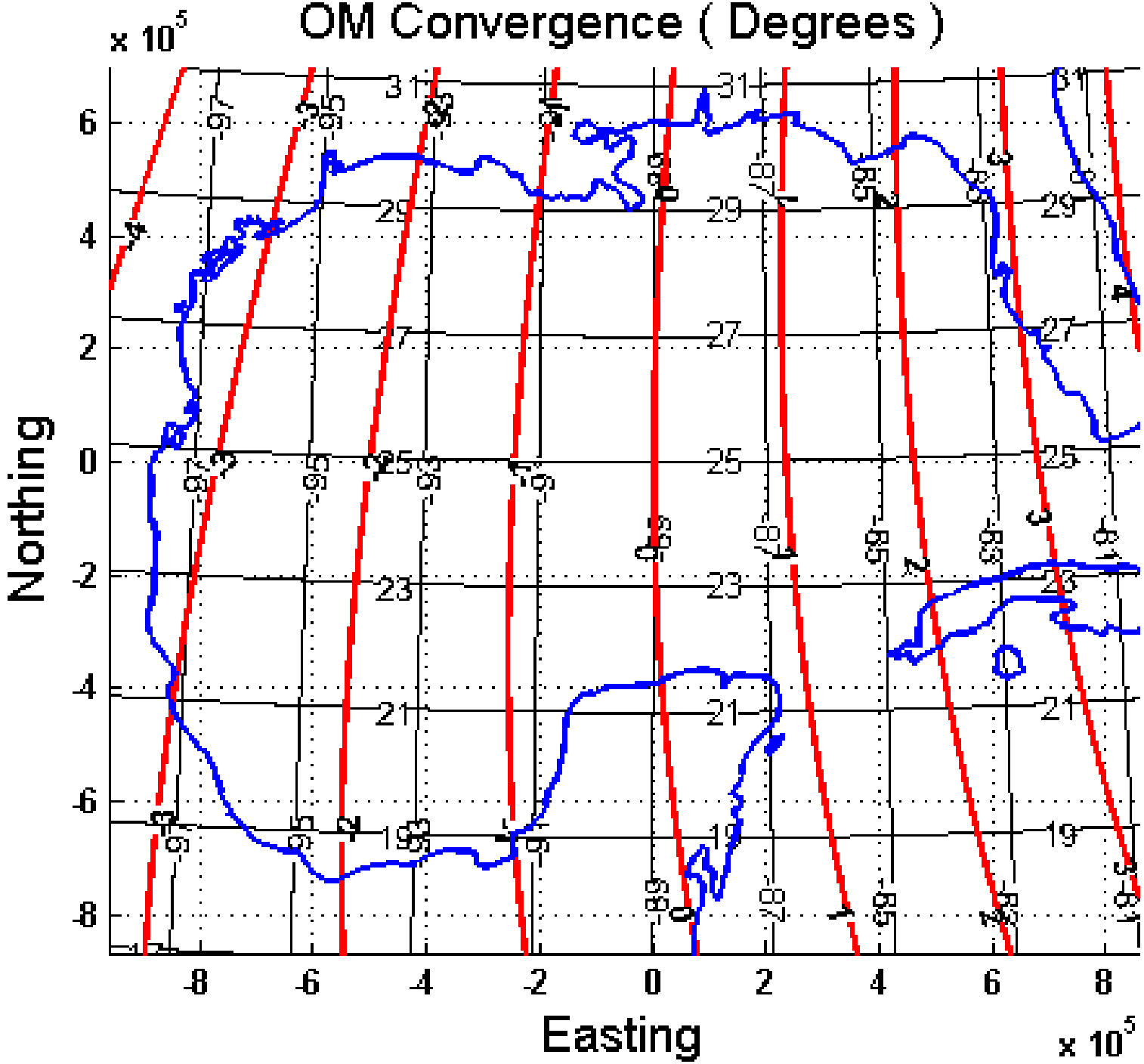

TM Latitude/Longitude Graticule
x 10^6
Northing
3.4
3.2
3
2.8
2.6
2.4
2.2
2
0
2
4
6
8
10
12
14
16
Easting
x 10^5
In Transverse Mercator the graticule of meridians (N-S) and parallels (E-W) are curves, but always orthogonal in any conformal projection

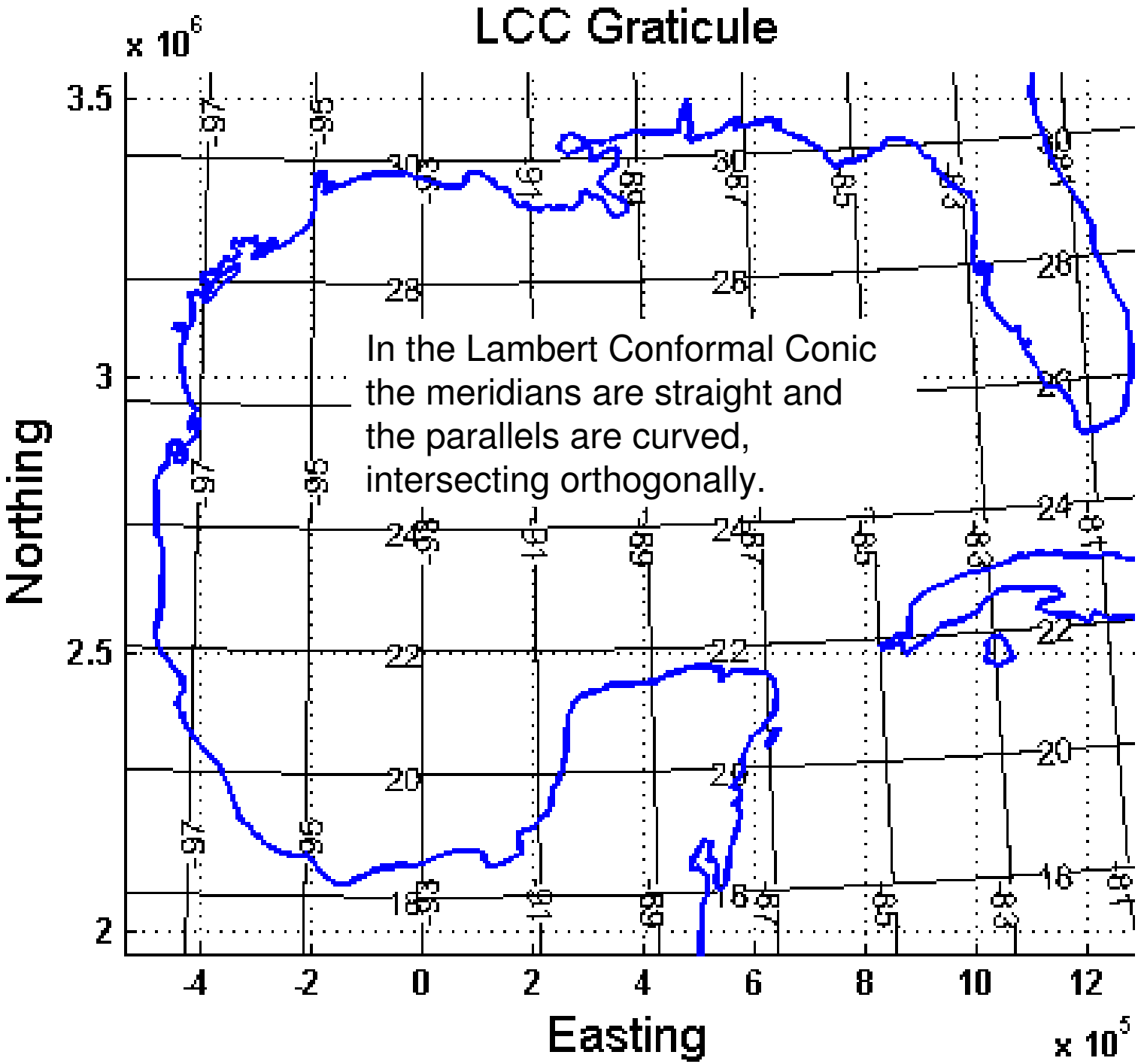

TM Point Scale Factor
x 10^6
Northing
3.4
3.2
3
2.8
2.6
2.4
2.2
2
0
2
4
6
8
10
12
14
16
Easting
x 10^5

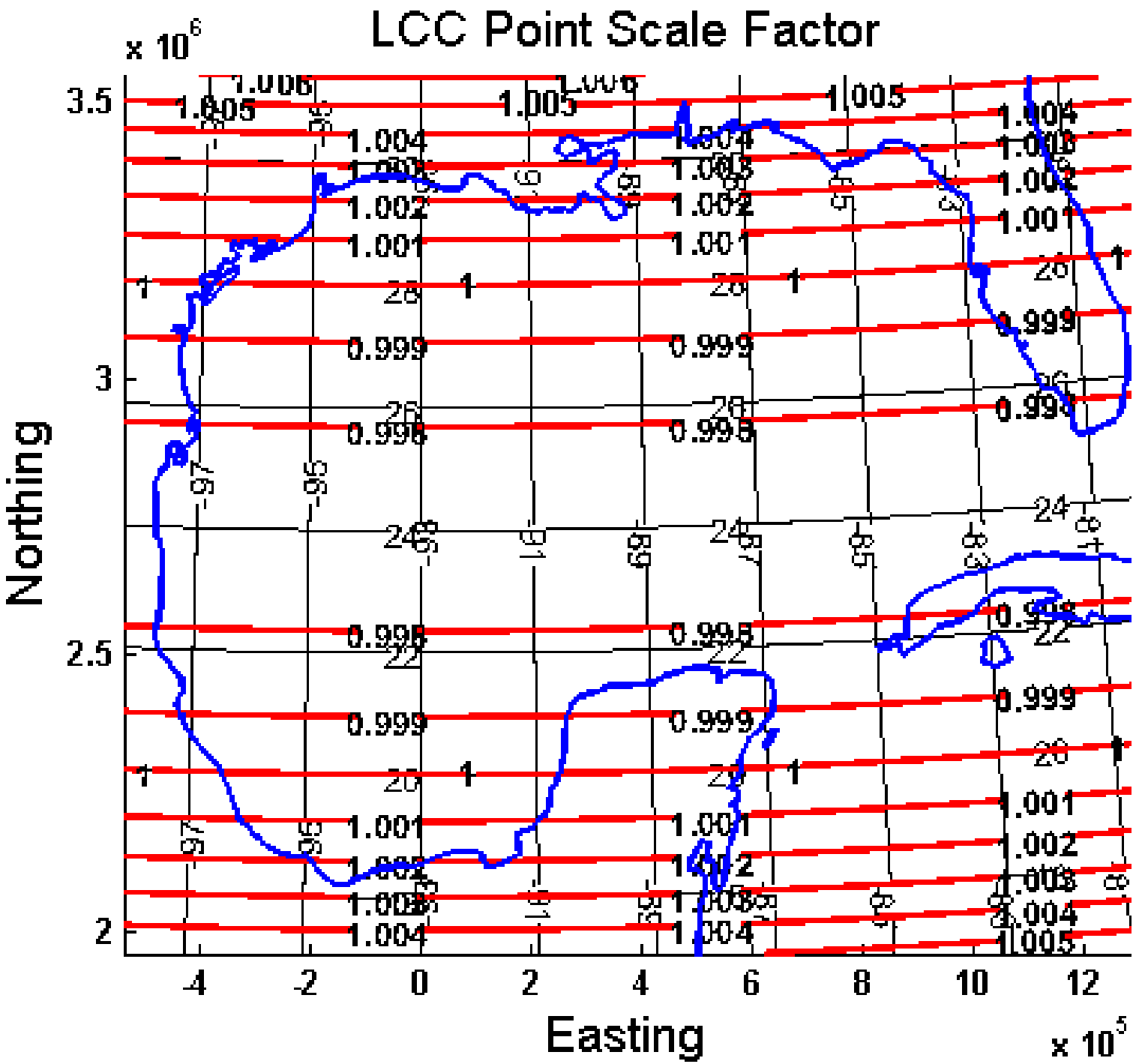

TM Convergence ( Degrees )
x 10^6
Northing
3.4
3.2
3
2.8
2.6
2.4
2.2
2
0
2
4
6
8
10
12
14
16
Easting
x 10^5

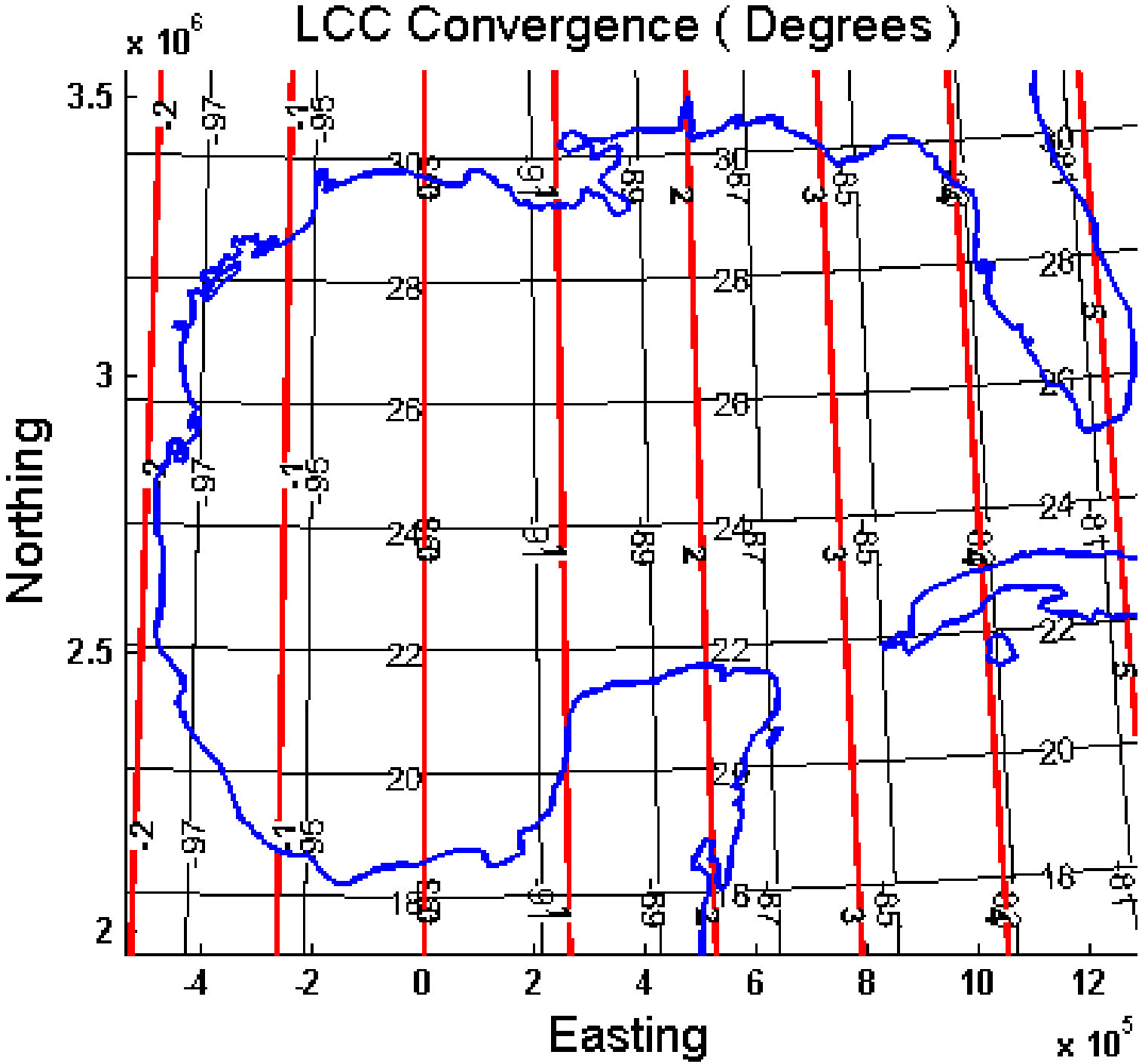

OM Latitude/Longitude Graticule
x 10^5
Northing
Easting
x 10^5
The Oblique Mercator is for prospects with a long axis at an angle to the graticule. That doesn't apply to the GOM.

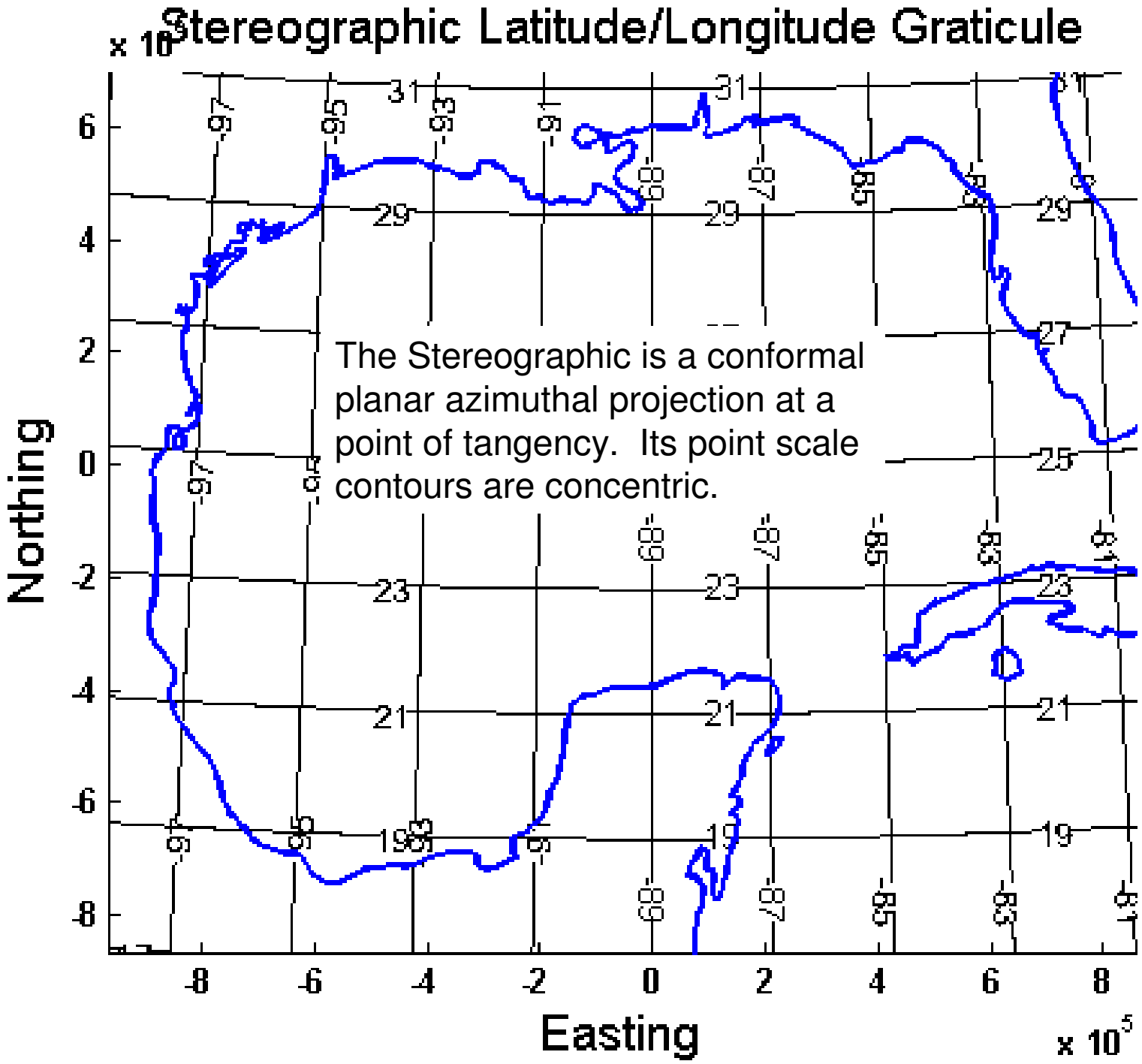

OM Point Scale Factor
x 10^5
Northing
Easting
x 10^5
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
1.012
1.01
1.008
1.006
1.004
1.002
1
31
29
27
25
23
21
19

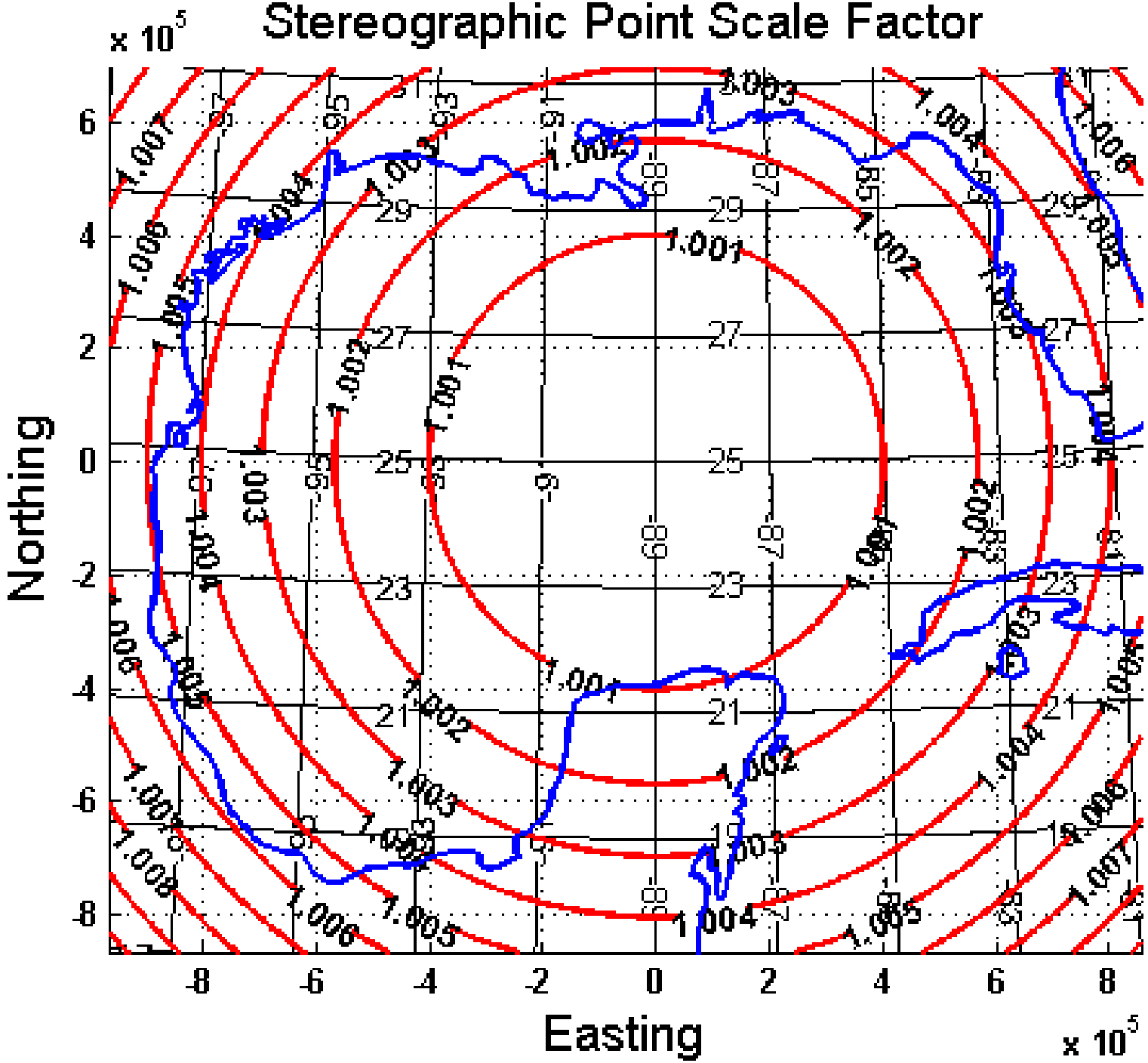

OM Convergence ( Degrees )
x 10^5
Northing
Easting
x 10^5
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
31
29
27
25
23
21
19

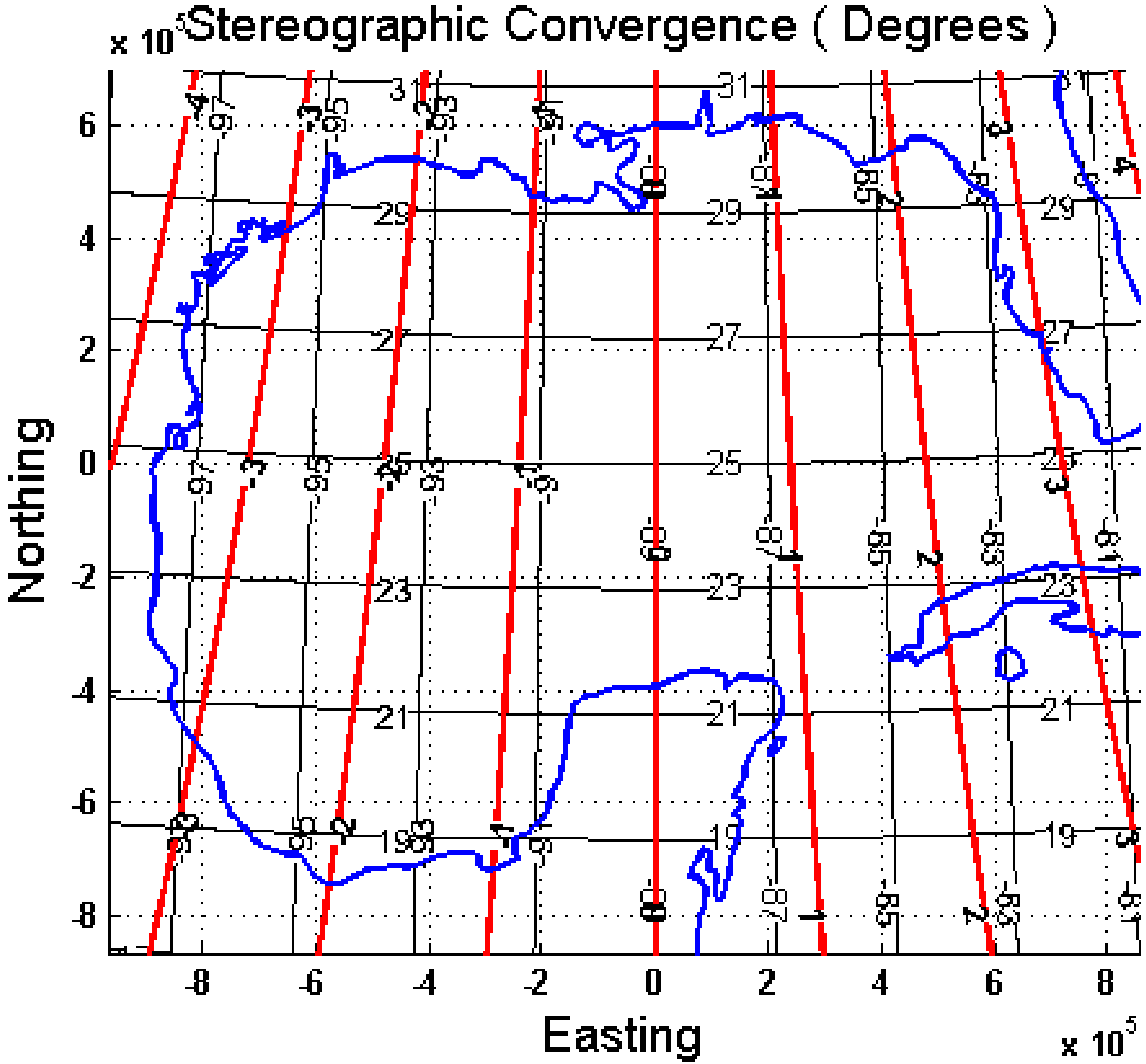

LCC Graticule
x 10^6
3.5
3
2.5
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5
In the Lambert Conformal Conic
the meridians are straight and
the parallels are curved,
intersecting orthogonally.

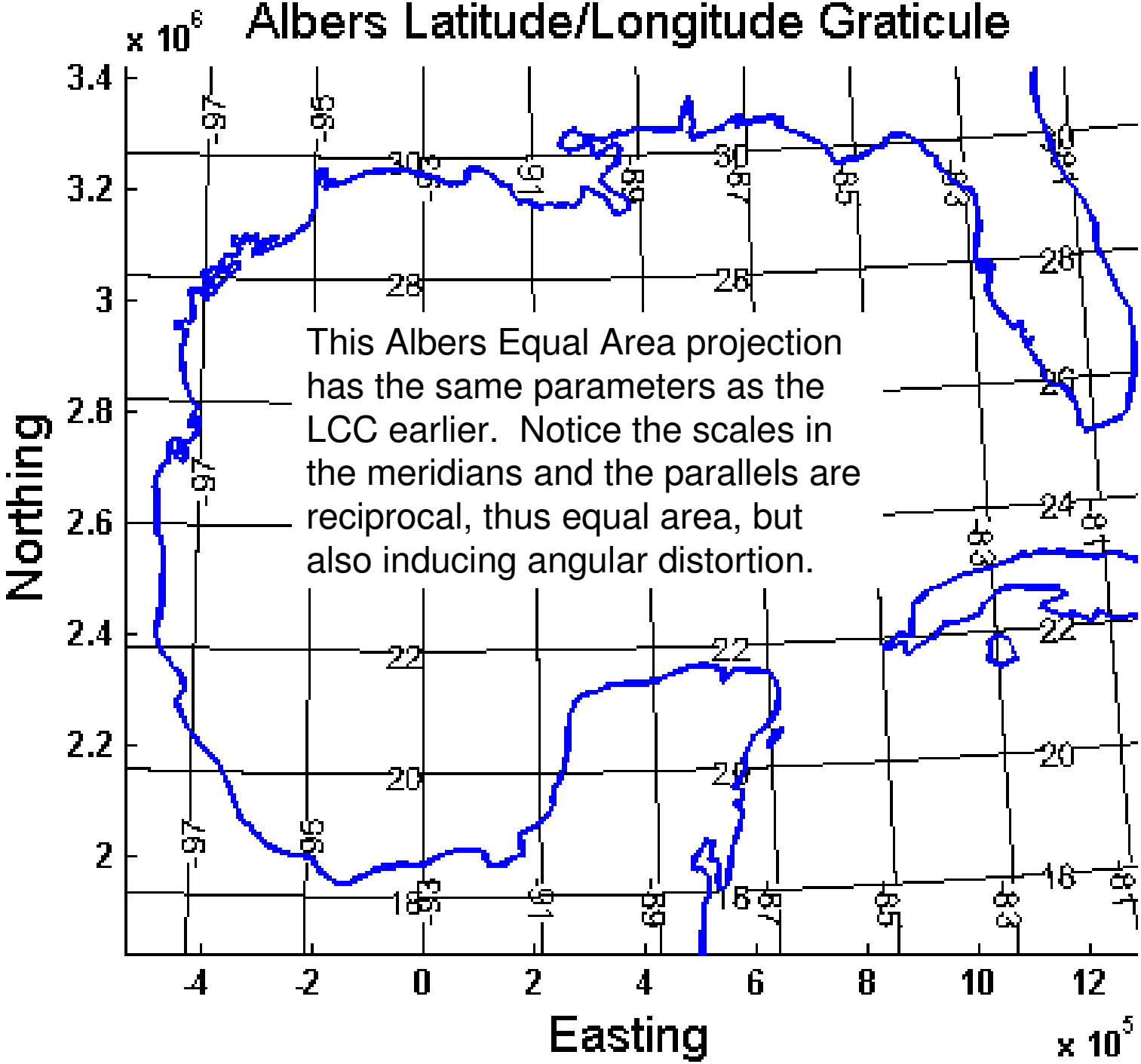

LCC Point Scale Factor
x 10^6
3.5
3
2.5
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5
1.006
1.005
1.004
1.003
1.002
1.001
1
0.999
0.998
0.997
0.996
0.995

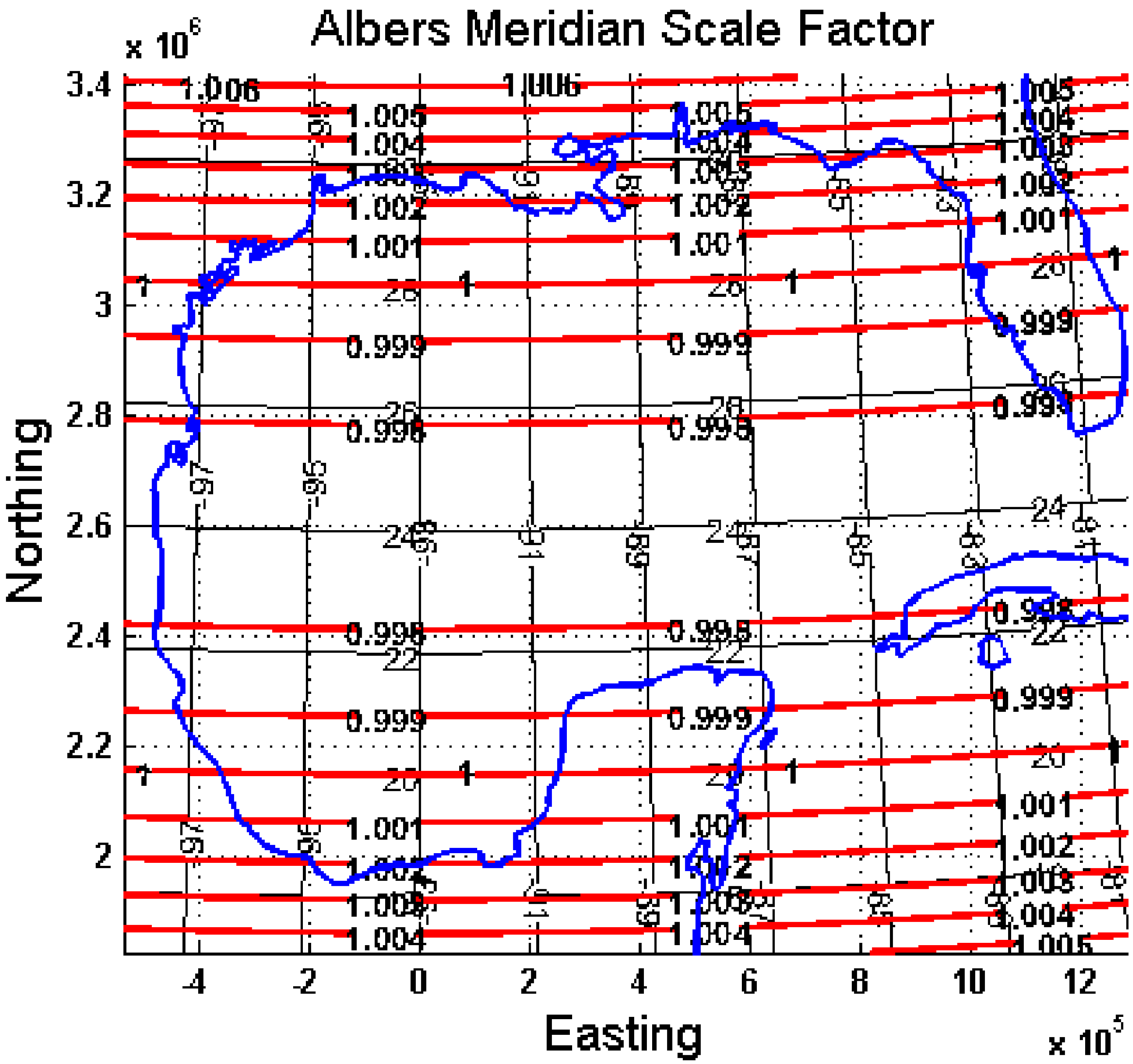

LCC Convergence ( Degrees )
x 10^6
3.5
3
2.5
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5

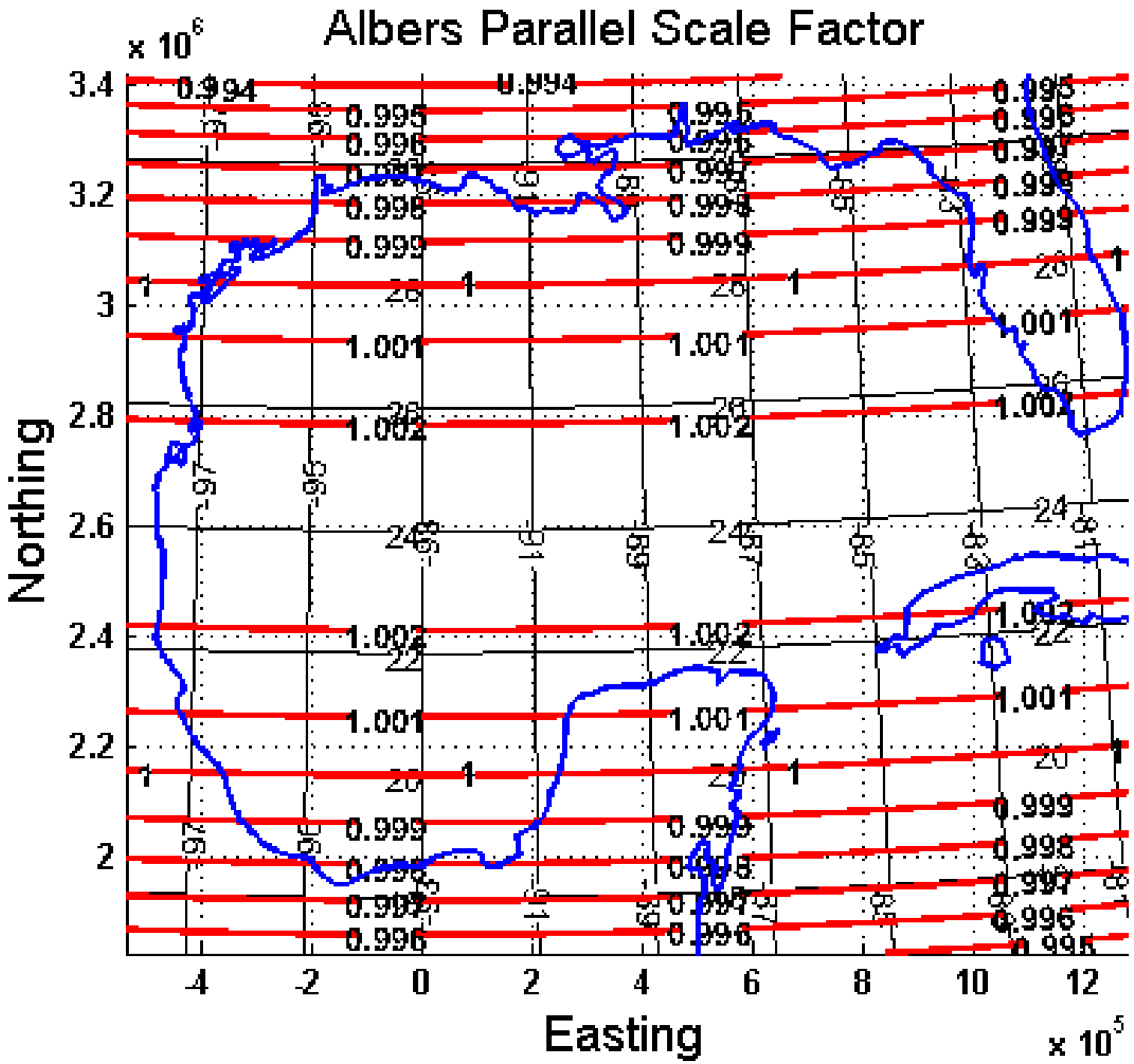

Stereographic Latitude/Longitude Graticule
x 10^5
Northing
Easting
x 10^5
The Stereographic is a conformal planar azimuthal projection at a point of tangency. Its point scale contours are concentric.

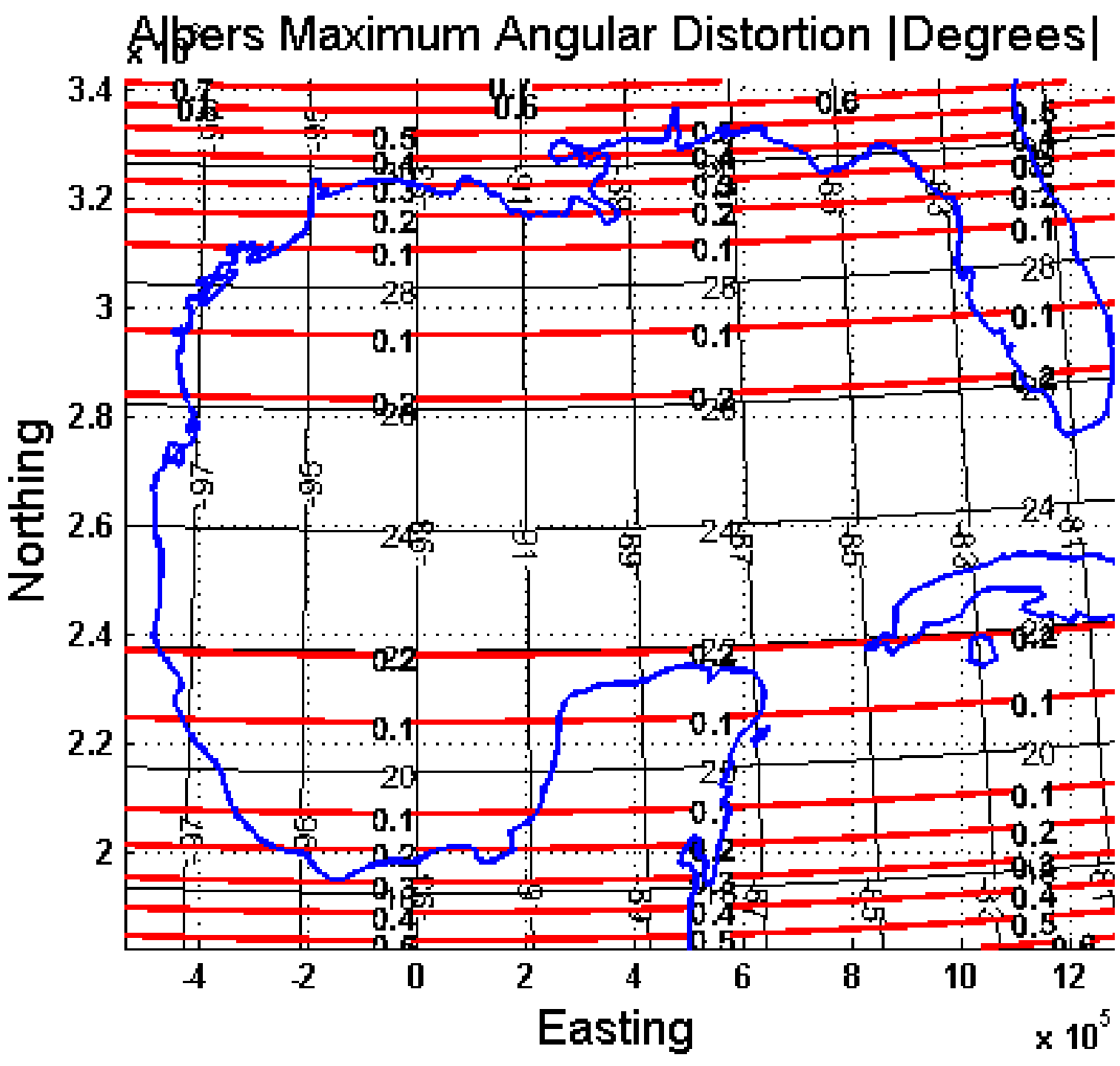

Stereographic Point Scale Factor
x 10^5
Northing
Easting
x 10^5
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
1.001
1.002
1.003
1.004
1.005
1.006
1.007
1.008
29
27
25
23
21
19

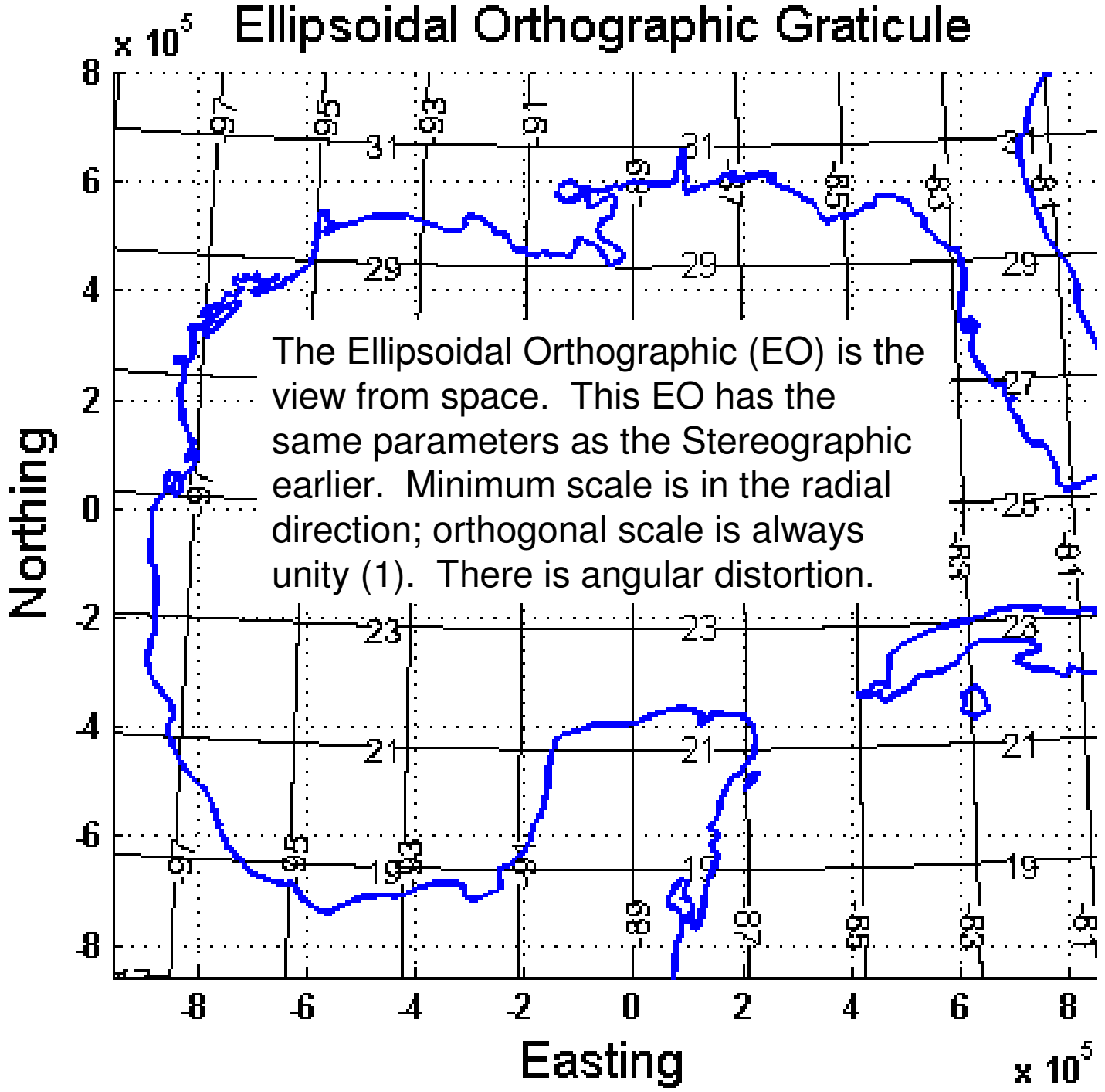

Stereographic Convergence ( Degrees )
x 10^5
Northing
Easting
x 10^5
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
31
29
27
25
23
21
19

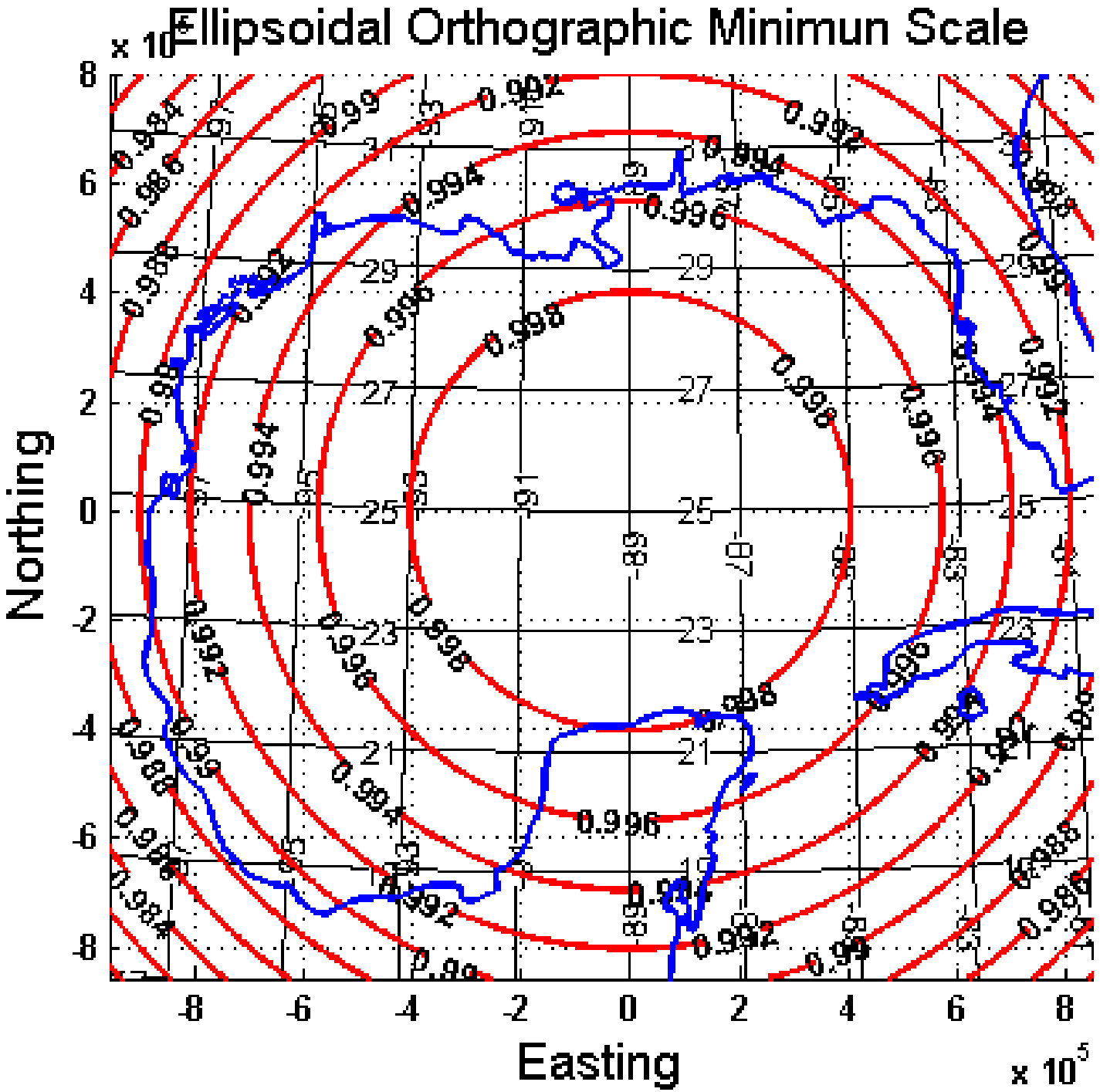

Albers Latitude/Longitude Graticule
x 10^6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5
This Albers Equal Area projection has the same parameters as the LCC earlier. Notice the scales in the meridians and the parallels are reciprocal, thus equal area, but also inducing angular distortion.

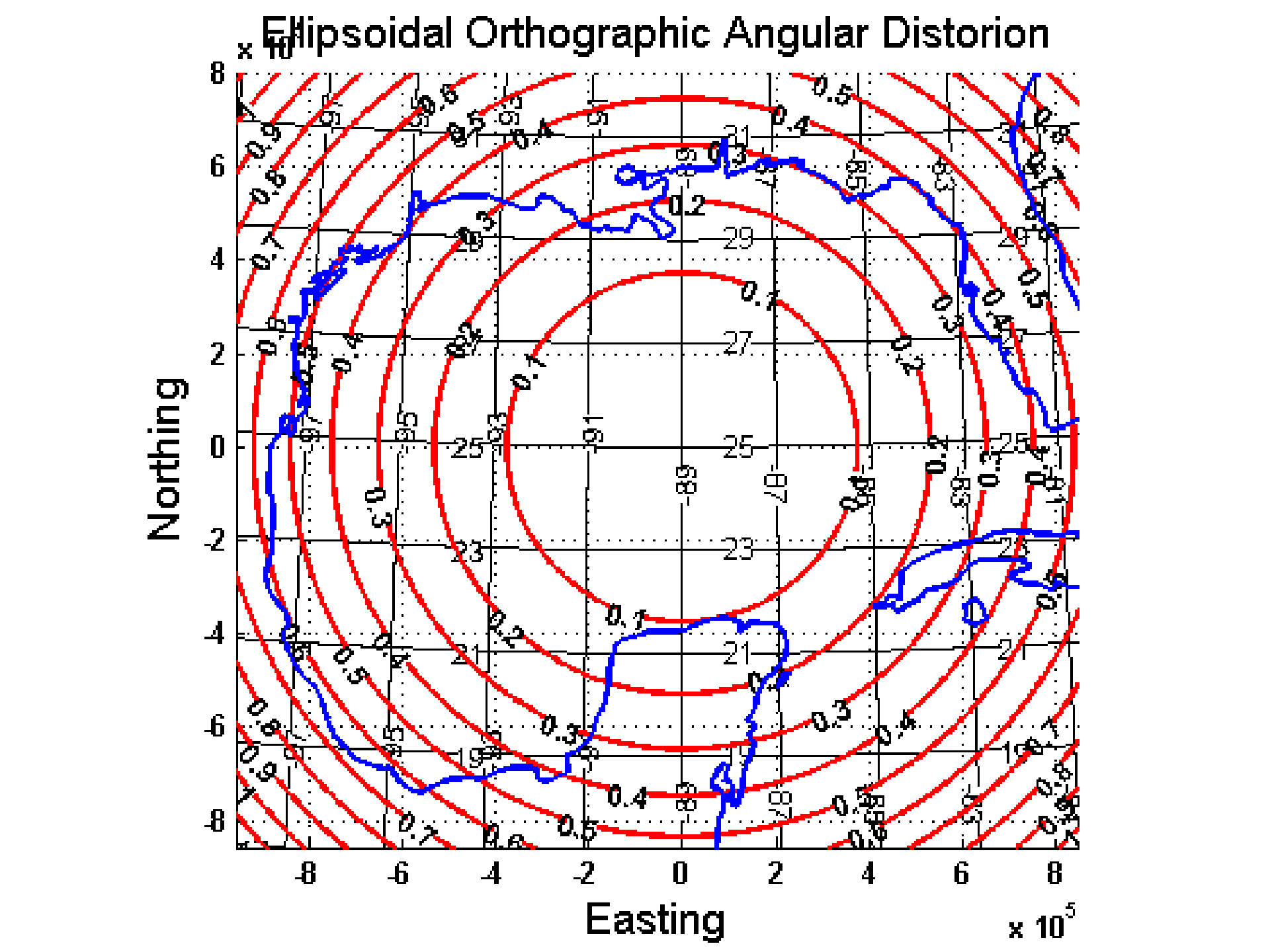

Albers Meridian Scale Factor
x 10^6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5
1.006
1.005
1.004
1.003
1.002
1.001
1
0.999
0.998
0.997

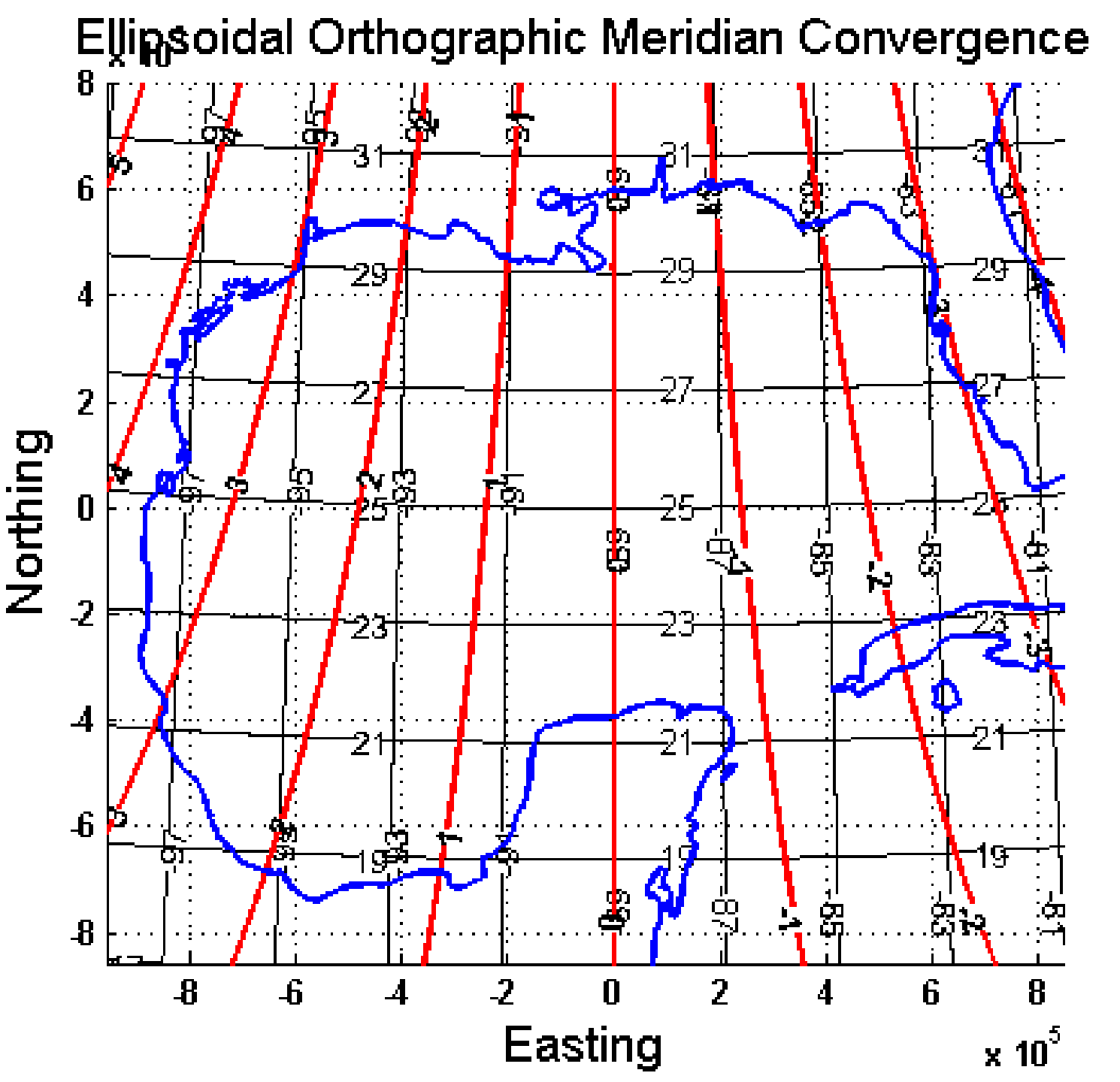

Albers Parallel Scale Factor
x 10^6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5
0.994
0.995
0.996
0.997
0.998
0.999
1
1.001
1.002


Albers Maximum Angular Distortion |Degrees|
x 10^6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Northing
-4
-2
0
2
4
6
8
10
12
Easting
x 10^5


Ellipsoidal Orthographic Graticule
x 10^5
Northing
Easting
x 10^5
8
6
4
2
0
-2
-4
-6
-8
The Ellipsoidal Orthographic (EO) is the view from space. This EO has the same parameters as the Stereographic earlier. Minimum scale is in the radial direction; orthogonal scale is always unity (1). There is angular distortion.


Ellipsoidal Orthographic Minimun Scale
x 10^6
Northing
Easting
x 10^5
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
0.992
0.994
0.996
0.998
29
27
25
23
21
19


Ellipsoidal Orthographic Angular Distorion
x 10^6
Northing
Easting
x 10^5
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8


Ellipsoidal Orthographic Meridian Convergence
x 10^6
8
6
4
2
0
-2
-4
-6
-8
Northing
-8
-6
-4
-2
0
2
4
6
8
Easting
x 10^5

# Cartographic Characteristics, 1 of 2

- Plate Carrée
  - Not a projection
  - The latitude/longitude, meridian/parallel graticule is the grid
  - No Northings or Eastings, no linear unit (such as meters)
- Mercator
  - Cylindrical axis is the spin axis
  - Meridians and parallels are straight
  - Scale increases north and south
  - No convergence
- Transverse Mercator
  - Cylindrical axis is in the Equatorial plane
  - Meridians and parallels are curved
  - Scale increases east and west
- Oblique Mercator
  - Cylindrical axis is oblique (at an angle to the graticule)
  - Meridians and parallels curved
  - Scale increases away from the central azimuth

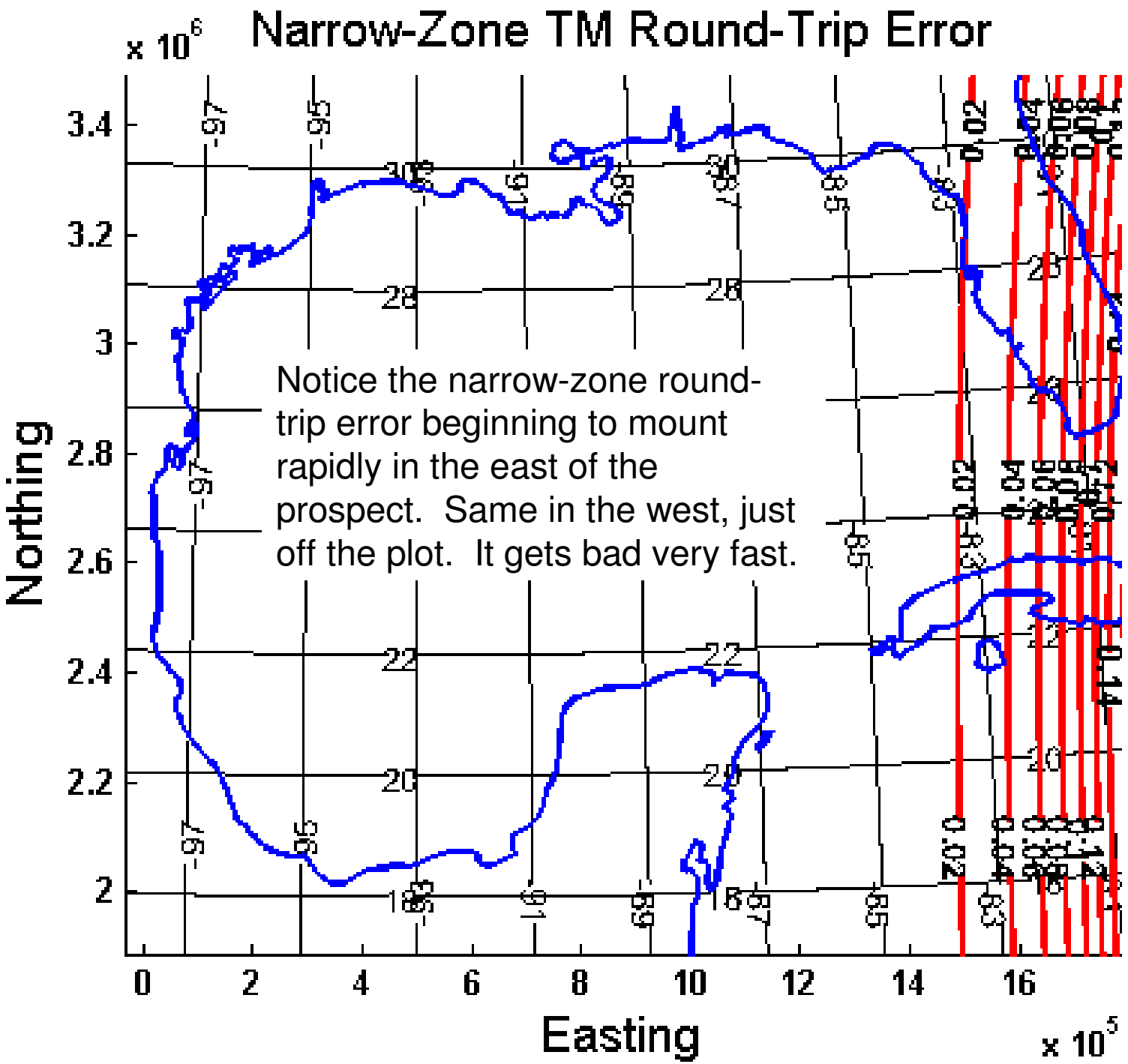
# Cartographic Characteristics, 2 of 2

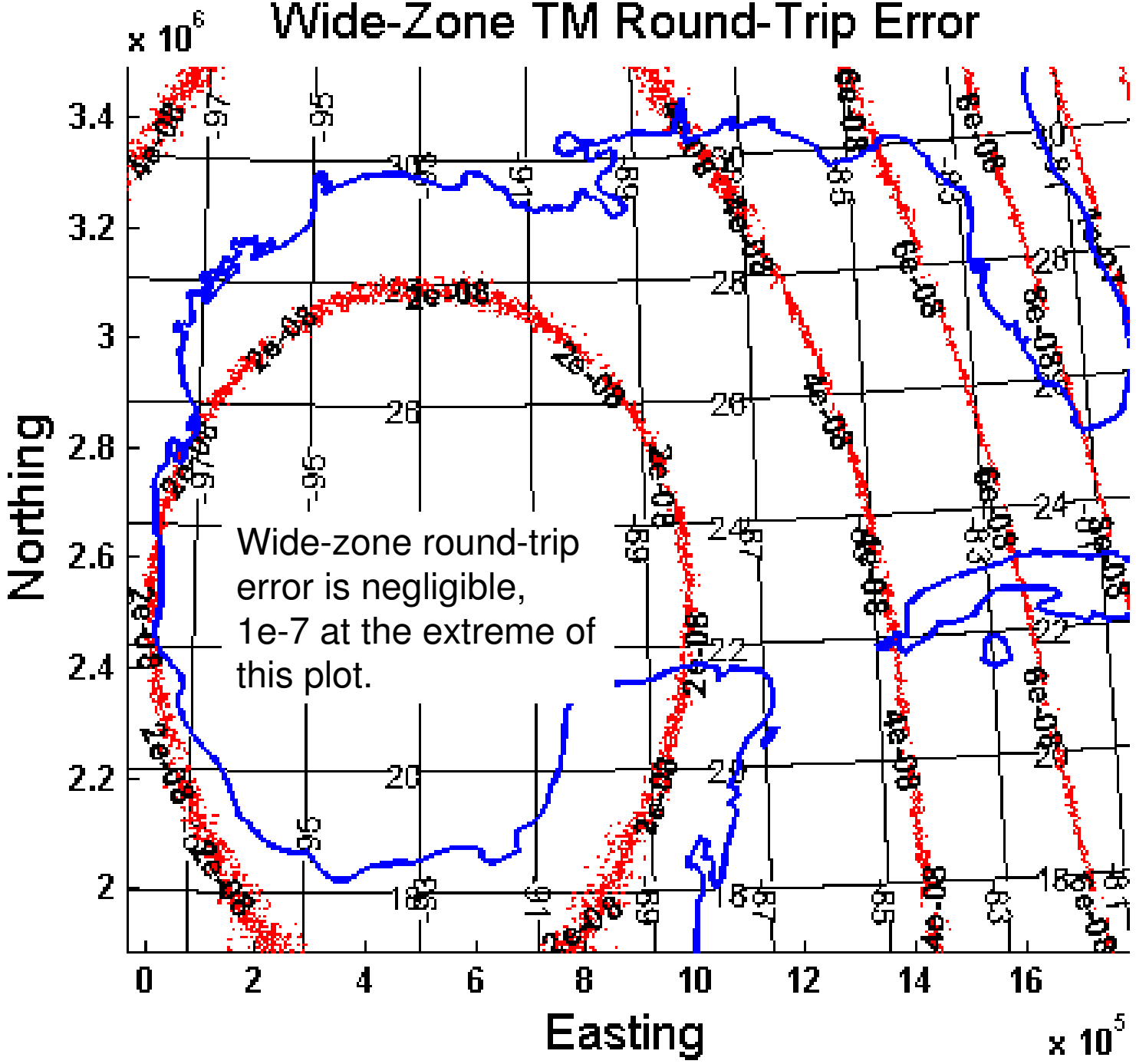
- Lambert Conformal Conic
  - Conical axis is the spin axis
  - Meridians straight and parallels curved
  - Scale increases north and south
- Stereographic
  - Azimuthal plane
  - Meridians and parallels curved
  - Scale increases circularly
- Albers Equal Area
  - Conical axis is the spin axis
  - Orthogonal scales at a point are reciprocal
  - Thus, the equal-area property
- Ellipsoidal Orthographic
  - Azimuthal plane
  - The view from space, not conformal, not equal area
  - Scale in the circular direction always unity (1)

# Afterword on Transverse Mercator

TM (Gauss-Kruger) algorithms approximate elliptical integrals with infinite series expansions that are necessarily truncated. The earliest TM implementations (Redfearn, Thomas, Snyder, US Department of the Army, which “invented” the UTM grid system) truncated severely to suit the computational equipment of the day. Collectively, these are called “narrow-zone” algorithms because the round-trip error begins to mount annoyingly about 4 to 5 degrees from the CM. New, wide-zone implementations are stable 40 degrees from the CM, some as many as 90 degrees.

But since many TM implementations are “narrow zone”, another potential distortion is narrow-zone round-trip error when working outside of the width of a UTM zone. The following two plots show narrow-zone and wide-zone round-trip error.


Narrow-Zone TM Round-Trip Error
x 10^6
Northing
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Notice the narrow-zone round-trip error beginning to mount rapidly in the east of the prospect. Same in the west, just off the plot. It gets bad very fast.
0
2
4
6
8
10
12
14
16
Easting
x 10^5


Wide-Zone TM Round-Trip Error
x 10^6
Northing
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Wide-zone round-trip error is negligible, 1e-7 at the extreme of this plot.
0
2
4
6
8
10
12
14
16
Easting
x 10^5

# Concluding Comments

- This brief presentation is only a sampler.  It cannot be exhaustive.
- Several useful projections and many characteristics are not covered
- Specific projects require specific projections and specific parameters for those projections
- Consult with a geodesist if in doubt about what's best for your project
- LCC and Albers are conic projections with the same parameters, but different results
- Stereographic and Orthographic are planar azimuthal projections with the same parameters, but different results
- More information on narrow-zone and wide-zone TM can be found here: http://www.hydrometronics.com/downloads/EPSG%20Wide-Zone%20TM%20r2.pdf
- More information on Ellipsoidal Orthographic can be found here: http://www.hydrometronics.com/downloads/Ellipsoidal%20Orthographic%20Projection.pdf